Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

DERIVATION AND MODELLING OF DYNAMIC EQUATIONS OF SYNCHRONOUS GENERATOR FOR HYDRO POWER PLANT

Babarinde Victor Olumuyiwa and Chizindu Stanley Esobinenwu
Department of Electrical/Electronic Engineering, University of Port Harcourt, Rivers State, Nigeria
Email: victormuyiwa@yahoo.com; chizindu.esobinenwu@uniport.edu.ng
DOI: https://doi.org/10.5281/zenodo.17242224

Abstract
This study focused on the derivation and modelling of dynamic equations for synchronous generator for hydro power plant. Hydro power plant mainly consists of three sections, governor (controller), hydro servo system and hydro turbine. The hydro turbine governor is usually coupled to a synchronous generator to drive the shaft so that the mechanical energy of turbine is converted to electrical energy. Accurate modeling of hydraulic turbine and its governor system is essential to depict and analyze the dynamic system response. In this work, both hydraulic turbine and turbine governor system were modeled. The hydro turbine model is designed using penstock and turbine characteristic equations. The simulation model is developed using MATLAB/SIMULINK. The dynamic response of the governing system to the disturbances such as load variation on the generator parameter during fault was presented. The results graphically demonstrate the effect of load variation on generator parameters under a three phase to ground fault. The transient behavior of generator voltage, current and the rotor speed are also captured. Similarly, two hydropower dams along the River Niger (Kainji and Jebba dams) in Nigeria were analyzed for energy generation using multilayer perceptron artificial neural network. Total monthly historical data of Kainji and Jebba hydropower reservoirs' variables and energy generated were collected for a period of forty-two years (1980-2021) and (1994-2021) for the network training. These data were divided into training, testing and holdout data set. The neural network analysis yielded a good forecast for Kainji and Jebba hydropower reservoirs with correlation coefficients of 0.89 and 0.77 respectively. These values of the correlation coefficient showed that the networks are reliable for modeling energy generation as a function of reservoir variables for future energy prediction.

Keywords: *Derivation, Dynamics, Synchronous, Genarator and Hydro Power Plant.*

Introduction

As hydropower generation accounts for 75% of renewable sources in the world's electrical mix (Turgeon *et al.*, 2021), optimizing hydropower generation is crucial from an economic and environmental perspective. An optimized hydropower operation offers numerous benefits, including better use of water resources, increased renewable energy production, and mitigating growing energy demand. To achieve this optimization, accurate inflow prediction is essential for hydropower generation forecasting, which involves estimating the power available to the grid using hydrological and climatic data. By adopting performance criterion and optimizing dispatch of generating units, hydropower plants can operate at their best efficiency, ultimately providing a reliable and cost-effective solution to meet rural energy demands (Uhunmwangho *et al*., 2010). A hydropower plant's operation involves complex processes, including pre-operation, real-time, and post-operation stages. The pre-operation steps can be separated into long-term, medium-term, short-term, and real-time scheduling (Bordin *et al*., 2020). Proper planning is essential, as hydropower offers the lowest cost among renewable energy sources, with low risk and a longer life span of up to 80 years (MAC, 2012). Major decisions are made during the design process, and system design tradeoff studies are carried out to select an optimum design (Black, 98). Considerations in turbine design and plant operation are critical in achieving a viable hydroelectric system, which relies on hydro turbines to transfer kinetic energy from moving water to a rotating shaft, generating electricity.

Hydro-turbines operate by passing water through a row of blades attached to a rotating shaft, generating rotational motion that is transferred to a generator to produce electricity. Various turbine types are designed to maximize output in specific situations, selected based on hydraulic head and hydroelectric discharge. Water enters these turbines radially or perpendicularly to the rotational axis. Once entering, the water flows inwards, towards the center. Management of hydro turbines, typically installed in dams or reservoirs, involves assessing water flow through the turbine, categorized into axial, radial, or mixed flows. The Francis turbine, a reaction turbine used in medium- or large-scale hydroelectric plants, is beneficial due to its versatility in handling various head levels (2-300 meters) and orientations (horizontal or vertical). Francis turbines lose pressure but stay at most less same speed. Once the water has flown through the turbine, it exists axially- parallel to the rotational axis. Proper turbine selection and management are crucial for efficient and effective hydroelectric power generation, providing a renewable and clean source of energy.

Statement of the Problem

Hydroelectric plants are especially vulnerable to climate change and its resulting effects, such as extreme weather events like hurricanes and droughts. The future of hydroelectricity lies in the development of better technologies improving its efficiency, as well as in energy decentralization, building smaller, interconnected, and distributed hydroelectric plants equipped with battery storage. The statement of the problem is embedded in the followings:

i. **Cavitation:** Which occurs due to the formation of the voids or bubbles where the pressure of the liquid changes rapidly. The bursting of such voids can cause strong shockwaves as a result of change in fluid pressure. Cavitation defects can be found at four places: leading edge, trailing edge, draft tube swirl and inter blade vortex.

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

ii. **Erosion:** Erosion problem which reduces power generation. Erosion rate depends upon the silt size, concentration, and hardness, velocity of striking particles with components and material of component.

iii. **Fatigue:** This is a process which makes material weak by repeated cyclic stress. It is another problem which can become cause of turbine failure in case of hydro turbine assembly connected with number of components and welding joints. This process is noticed during the transition of loud variation in hydro turbine and material of runner and other component.

iv. **Material defect:** Maximum failure in hydro turbine is due to cavitation and silt erosion. Materials defects in turbines and other components are controlled during manufacturing, but at the time of installation some material defects can also generate causing failure of components.

Aim and Objectives of the Study

This study is aimed at dynamic assessment of hydro turbines for effective power utilization. The specific objectives of the study are to:

i. Derive and model the dynamic equations of a synchronous generator

ii. Evaluate through simulation the modeled equations of the hydro-power plants and generator under ideal condition and under a three phase to ground fault.

Conceptual Review

Hydroelectric power is a renewable energy source that harnesses the energy of water in motion. This energy can be seen as a form of solar energy, as the sun powers the hydrologic cycle, which gives the Earth its water (Salihu, 2025). The hydrologic cycle is a complex process that involves the continuous movement of water on, above, and below the surface of the Earth. The hydrologic cycle begins with evaporation, where water from the oceans, lakes, and rivers is heated by the sun and turns into water vapor. This water vapor rises into the atmosphere and cools, condensing into clouds. When the clouds become saturated with water, they release their water content in the form of precipitation, which can be in the form of rain, snow, sleet, or hail (dos Santos Sousa et al., 2025). Once the precipitation reaches the Earth's surface, it can follow several paths. Some of it may evaporate, while some of it may percolate into the soil to become groundwater. Groundwater can eventually feed into streams, rivers, and lakes, where it can evaporate again and continue the cycle (Zvereva et al., 2025). The movement of water in the hydrologic cycle is what drives hydroelectric power generation. Hydroelectric power plants harness the energy of moving water by channeling it through a turbine, which converts the kinetic energy of the water into electrical energy (Liu et al., 2025).

Turbine Efficiency

No system can achieve 100% efficiency, as some of the supplied energy is inevitably lost due to friction in the moving parts. This is particularly true for turbines, which convert hydraulic energy into mechanical energy. Turbine efficiency is a measure of how effectively this conversion process occurs. Turbine efficiency is defined as the ratio of the power supplied by the turbine shaft to the available hydraulic power.

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

In other words, it measures the proportion of the available energy that is actually converted into useful power. The remaining energy is lost as heat, noise, or vibration due to friction and other inefficiencies. For example, if a turbine has an efficiency of 90%, it means that 90% of the available hydraulic energy is converted into mechanical energy, while the remaining 10% is lost as heat or other forms of energy. Thus, turbine efficiency is the ratio of the power supplied by the turbine shaft to the available hydrau lic power.

$\eta_t = \frac{P_m}{P_h}$

Where P_m – Turbine shaft power, P_h- available hydraulic power

Head measurement is a critical parameter in determining the efficiency of a turbine. In the context of hydropower generation, head refers to the vertical distance between the water source and the turbine. Accurate head measurement is essential to determine the available energy that can be harnessed by the turbine.

Turbine Power

Turbine power refers to the mechanical energy generated by a turbine, typically used to drive an electrical generator or other mechanical device. The power output of a turbine is determined by the energy transferred from the fluid (such as water or steam) to the turbine blades. In the context of hydropower generation, the turbine power is directly related to the energy available in the water. The water path, which refers to the route that the water takes as it flows through the turbine, plays a crucial role in determining the turbine power. The water path can affect the turbine power in several ways. For example, a longer water path can result in greater energy losses due to friction and turbulence, which can reduce the turbine power. On the other hand, a well-designed water path can help to minimize energy losses and maximize the turbine power. In addition, the water path can also affect the efficiency of the turbine. A turbine with a well-designed water path can operate more efficiently, resulting in higher power output and lower energy losses. Irrespective of the water path to the turbine, power generated by a turbine is given as

Power. $P_t = \rho\ Q\ H_n\ g\ \eta_t$

P_t– Power generated by turbine shaft (watts)

H_n– net head (m)

Q – discharge (m^3/s)

η_t – turbine efficiency (normally 80-90%)

For impulse turbines, the reduction in head is as a result of the gross head measured from the point of impact of the water jet, normally above the downstream water level. As compared with reaction turbines performance, the different is not negligible in low head schemes. The turbine efficiency is the ratio of the mechanical power transmitted by turbine shaft to the hydraulic power equivalent proportional to the measured floe rate under the net head.

Turbine Speed

Turbine speed refers to the rotational speed of a turbine, typically measured in revolutions per minute (RPM). Turbine speed is a critical parameter in determining the efficiency and power output of a turbine.

The turbine speed is influenced by several factors, including the head (pressure) of the fluid, the flow rate of the fluid, and the design of the turbine. In general, the turbine speed increases with increasing head and flow rate. In the context of hydropower generation, the turbine speed is directly related to the energy available in the water. The water path, which refers to the route that the water takes as it flows through the turbine, plays a crucial role in determining the turbine speed. The water path can affect the turbine speed in several ways. For example, a longer water path can result in greater energy losses due to friction and turbulence, which can reduce the turbine speed. On the other hand, a well-designed water path can help to minimize energy losses and maximize the turbine speed. In addition, the turbine speed can also affect the efficiency of the turbine. A turbine operating at its optimal speed can achieve higher efficiency and power output, while a turbine operating at a speed that is too high or too low can experience reduced efficiency and power output. In hydropower applications, the turbine speed is typically optimized to match the available head and flow rate of the water. This can involve adjusting the turbine's blade angle, pitch, or other design parameters to achieve the optimal speed. Outside the inertia provided by the fly wheel on the generator or turbine shaft, Additional inertia is required to regulate the turbine speed.

For a rotating system.

$$\frac{dw}{dt} = \frac{P_t - P_l - Bw^2}{jw}$$

P_t = Turbine power (watt)
P_l = Load power (watt)
w = turbine speed in (rad/sec.)
B = Frictional torque coefficient of turbine and generator (N.m(rad/sec.))
j = Moment of inertia of the rotating system (kg/m^2)

Operation is steady when $P_t = P_l + B * w^2$, $\frac{dw}{dt}$ = 0, and w = constant. When turbine output power is greater than or less than $P_l + B * w^2$, the governor adjusts the system such that turbine output power equals generator output power. This results in a first order differential equation.

$$w = \sqrt{\frac{P_t - P_l}{B} 1 - e^{-\frac{2B}{j}t} + w_0^2 * e^{-\frac{2B}{j}t}} \quad .$$

where the turbine speed N is given as

$$N = \frac{60 * w}{2\pi} \text{ (r.p.m)}$$

Speed control is done by the speed governor. The mechanism involves adjustment of the guide vane to regulate the discharge through the turbine. The determining factor is the load. In single operation with less load, the output adjustment is dependent on load variation. This controls the frequency. When the operation stops due to fault or breakdown, the turbine immediately shuts down, thus closing the guide prevent abnormal rising of turbine and generator speed (JP Design 2011).

Head Losses

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

In the context of hydropower generation, head losses refer to the loss of energy that occurs as water flows through a system, such as a penstock, turbine, or pipeline. Head losses can be caused by various factors, including friction, turbulence, and changes in flow direction or velocity. There are several types of head losses that can occur in a hydropower system, including:

1. Friction head loss: This type of head loss occurs due to friction between the water and the pipe or channel walls.
2. Minor head loss: This type of head loss occurs due to turbulence and changes in flow direction or velocity, such as at bends, valves, or other fittings.
3. Major head loss: This type of head loss occurs due to significant changes in flow direction or velocity, such as at the inlet or outlet of a turbine.

Head losses can have a significant impact on the efficiency and power output of a hydropower plant. As head losses increase, the available head (the difference in elevation between the water source and the turbine) is reduced, resulting in lower power output. The design head, on the other hand, is the head at which the turbine is designed to operate at its maximum efficiency. The design head is typically used to define the design power output of a turbine, which is the maximum power output for the best efficiency head.

In a hydropower station with a high head scheme, such as a long penstock, the variation in the water level can have less effect on power output than the variation of the head losses with the discharge. This is because the head losses can have a more significant impact on the available head and, therefore, the power output. For example, consider a hydropower plant with a design head of 500 meters and a penstock length of 1 kilometer. If the head losses in the penstock are 10% of the design head, the available head would be reduced to 450 meters. This would result in a lower power output, even if the water level remains constant. In contrast, if the water level varies by 10% due to changes in the water source or demand, the impact on power output would be less significant than the impact of the head losses. This is because the head losses would still be present, even if the water level remains constant. Therefore, it is essential to carefully consider the head losses in the design and operation of a hydropower plant, particularly in high head schemes with long penstocks. By minimizing head losses, hydropower plant operators can optimize power output and efficiency, while also reducing the environmental impact of the plant. When the reservoir water level changes during the powerplant operation, a maximum water level may appear when the weir discharges the maximum flood. The crest of the weir gives the full storage water level and the minimum exploitation water level is determined by requirements of the intake operation.

Estimation of Plant Capacity and Power Generation

The estimation of plant capacity and power generation will require the consideration of complex system dynamics and uncertainties. Simulation methods, which use computer simulations to estimate plant capacity and power generation, can provide accurate estimates if the simulation models are well-validated and the uncertain parameters are well-characterized. By using reliable data and modeling techniques, developers can make informed decisions about plant design, investment, and operation, and can ensure that the plant capacity and power generation estimates are accurate and reliable. For a given time period, power generated by a hydroelectric plant with z turbined of rated power P is given as

$$E = \sum_{j=1}^{z} \int_{0}^{T} P_j(t)dt$$

As for the plant factor, in application, a plant cannot be in operation for 8760 hours (a year). Thus, F is a function of the availability factor Δ and the plant's mean power coefficient ω

$$F = \Delta\ \omega$$

The availability of each of the units is function of the mean time between failure(MTBF) and mean time to repair(MTTR); given as

$$\Delta_y = \frac{MTBF}{MTBF + MTTR}$$

For n units in parallel operation, the availability factor is expressed as

$$\Delta = 1 - (1 - \Delta_y)^n$$

Advantages

1. No fuel requirement. That is no fuel is required to generate electricity. Hence, water source is perennially available.
2. Low runner cost. Electricity per KWH is very cheap as compared to thermal or nuclear
3. No problem of disposal of ash. Since no fossil fuel is used, so there is no problem of disposal of ash
4. Pollution free electricity generation. The electricity generated by hydropower does not produce any type of pollution
5. Simple in concept, self -contained and reliable in operation. The design concept of hydraulic power plants is simple in concept and the operation is self-contained and reliable as compared to thermal or nuclear plants.
6. Greater life expectancy. Modern hydraulic power plants have greater life expectancy of about 50years as compared to thermal or nuclear plants about 30years
7. Easily switched on and off in a short period. The hydraulic turbine is switched on and off in a very short period unlike thermal and nuclear plants.

Disadvantages

The following are disadvantages of hydroelectric power plants.

1. High capital cost. Hydroelectric power plants are capital intensive with a low rate of return.
2. Power dependent on the quantity of water available. Power generated by the hydro-plants is only dependent on the quantity of water available which in turn depends on the natural phenomenon of rain. The dry year is more serious for the hydro-electric project
3. Long erosion time. The completion of hydro-electric power plants takes a much longer period (about 10-15years) as compared to thermal power plants of (about 3years)

4. Site selection dependent on water availability. This is because the site criterion is dependent on availability of water on economical load. Such sites are usually away from load centers. Nevertheless, long transmission lines are highly needed to transmit power from station to consumers.

Presently, since national grid is now a reality, rivers are interconnected to aid power production and irrigation of flood control.

Related Works

Kokubu *et al*., (2012), in their work on performance improvement of a micro eco cross-flow hydro turbine, delve into the investigation of performance enhancements for a micro eco cross-flow hydro turbine, aiming to bolster the efficiency and effectiveness of small-scale hydro-turbine systems. The study focuses on optimizing the design and operational parameters of a micro eco cross-flow hydro turbine to maximize energy extraction while minimizing losses and environmental impact. By exploring innovative approaches and technologies tailored for micro-scale hydro-turbine systems, Kokubu *et al*. aimed to address the challenges associated with sustainable energy production at smaller scales. Their research contributes valuable insights into the optimization of micro hydro-turbines, paving the way for enhanced performance and broader adoption of small-scale hydroelectric power generation solutions. However, it should be noted that the empirical study by Kokubu *et al*. (2012) is related to objective (ii) "Analyze the hydro turbine input-output Modeling Estimation" as it investigates the performance enhancements for a micro eco cross-flow hydro turbine, focusing on optimizing design and operational parameters to maximize energy extraction while minimizing losses and environmental impact. The study's emphasis on analyzing the input-output relationships of the micro hydro-turbine system demonstrates the importance of understanding the complex interactions between turbine design, operational parameters, and energy output. The point of departure between the empirically reviewed work and the current study is the recognition that existing research, such as Kokubu *et al*.'s study, has primarily focused on optimizing the performance of small-scale hydro-turbine systems, whereas the current study aims to address the broader challenge of optimizing the assessment and management of hydro turbines in general, encompassing a wider range of turbine sizes and configurations.

Bahrami *et al*. (2016), in their research titled Multi-Fidelity Shape Optimization of Hydraulic Turbine Runner Blades Using a Multi-Objective Mesh Adaptive Direct Search Algorithm, concentrate on refining the design of hydraulic turbine runner blades through a multi-fidelity shape optimization approach employing a multi-objective mesh adaptive direct search algorithm. This research is centered on enhancing the efficiency and performance of hydraulic turbines by optimizing the geometric configuration of runner blades. By employing advanced computational techniques and optimization algorithms, the study aims to iteratively refine the blade shape to achieve multiple objectives, such as maximizing energy extraction, minimizing losses, and reducing flow disturbances. Through the integration of multi-fidelity modeling and adaptive search strategies, Bahrami *et al*. contribute to the advancement of optimization methodologies tailored specifically for hydraulic turbine applications. This research represents a significant step forward in the quest to enhance the efficiency and performance of hydraulic turbines, thereby facilitating the continued growth and sustainability of hydroelectric power generation.

In a related study, Li *et al*. (2018) carried out the analysis of pressure fluctuations in a prototype pump-turbine with different numbers of runner blades in turbine mode conduct a thorough analysis of pressure fluctuations occurring in a prototype pump-turbine, particularly focusing on the effects of different numbers of runner blades during turbine operation. This examination provides valuable insights into the intricate relationship between blade configurations and performance outcomes, offering essential knowledge for optimizing pump-turbine designs and enhancing overall operational efficiency and stability. Hence, the empirical study by Li *et al*. (2018) is related to objective (iv) "Evaluate through simulation the modeled equations of the hydro-power plants and generator under ideal condition and under a three-phase to ground fault" as it conducts a thorough analysis of pressure fluctuations in a prototype pump-turbine, which involves evaluating the performance of the turbine under different operating conditions. The study's focus on understanding the effects of different numbers of runner blades on pressure fluctuations demonstrates the importance of evaluating the performance of hydro-power plants under various conditions to optimize design and operation. The point of departure between the empirically reviewed work and the current study is the recognition that existing research, such as Li *et al*.'s study, has primarily focused on analyzing the performance of individual components of hydro-power systems, such as pump-turbines, under specific operating conditions, whereas the current study aims to develop a more comprehensive understanding of the dynamic behavior of hydro-power plants and generators under various fault conditions, including three-phase to ground faults.

He *et al*. (2020) delved into the examination of strategies for optimizing operation rules of cascade reservoirs in their study. Titled "Optimizing operation rules of cascade reservoirs for adapting to climate change," their research highlighted the pressing need for adaptive management practices in the face of climate change. By focusing on optimizing operation rules, He and his team aimed to enhance the resilience of hydroelectric power systems to climate variability and change. Their findings emphasized the importance of flexible management approaches to ensure the continued reliability and sustainability of hydroelectric power generation amidst changing environmental conditions. Soares *et al*. (2022) assessed the restoration from eutrophication in interconnected reservoirs using a model approach, providing insights into effective restoration strategies for optimizing water quality and ecological sustainability.

Methodology

This section which is usually materials and methods is introduced in order to explain the mathematical modeling and the dynamics of hydro turbine governor. Figure 3.1a shows a block diagram of hydro power plant while the simplified diagram is illustrated in Figure 3.1b. The stored water at certain head (H) contains potential energy. This energy is converted to kinetic energy. When it is allowed to pass through the penstock, this kinetic energy is converted to mechanical energy (rotational energy) which allows water to fall on the runner blades of the turbine. As the shaft of the generator is coupled to the turbine, the generator produces electrical energy by converting the mechanical energy into electrical energy. The speed governing system of turbine adjusts the generator speed based on the feedback signals of the deviations of both system frequency and power with respect to their reference settings. This ensures power generation at synchronous frequency.

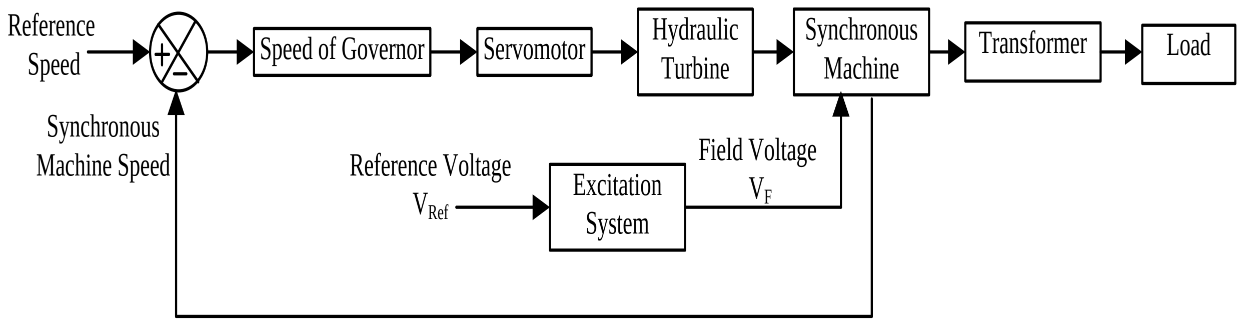

Figure 1: Block Diagram of a Hydro Power Plant

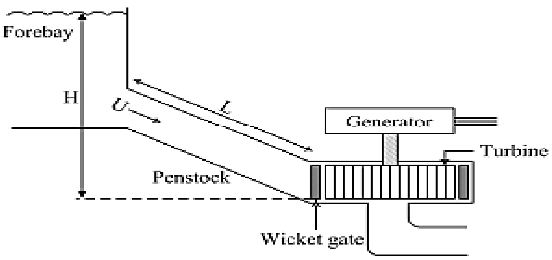

Figure 2: Simplified Block Diagram of a Hydro Power Plant

The hydro turbine governor is used to maintain a constant turbine speed hence the frequency and active power in response to load variation. The turbine governor regulates water input into a turbine, which in turn rotates the generator to produce electricity. This section details the mathematical modeling of the mechanical-hydraulic turbine governing system. A mathematical representation of a hydraulic governing system including the turbine-penstock is introduced here. Hydro turbine governing systems are strongly influenced by the effects of water inertia. To adjust the gate opening of the wicket gate, the servomotor controls a pilot valve. The servomotor is activated by the signals generated from the turbine governor. Equation 3.1 is derived based on the assumption of short penstock, insignificant water hammer and incompressible flow of fluid through penstock. It defines the characteristics of per unit turbine flow in terms of water time constant and head.

$$\frac{dQ}{dt} = \frac{(h_s - h - h_1)g \times A}{L} \qquad 1$$

Where:

Q = Per-unit turbine flow,

h_s = Static head of the water column,

h = Head at the turbine water admission,

h_l = Per-unit conduit head losses,

L = Length of the conduit section,
A = Cross-section area of the penstock,
g = Gravitational acceleration.
The given equation (in 1) can be converted to per unit equation by dividing it with its base quantity which results to equation above.

$$\frac{d\frac{Q_{base}}{Q_{base}}}{dt} = \frac{d\bar{q}}{dt} = \frac{(h_s - h - h_1)}{h_{base}} \times \frac{h_{base} \times gA}{L \times Q_{base}} = (h_s - h - h_1) \times \frac{1}{T_w}$$

$$T_w = \frac{L \times Q_{base}}{h_{base} \times gA}$$

Where:
h_{base} = Difference between Lake Head and Tail head.
Q_{base} = Maximum gate opening of the reservoir. $\bar{q}$ is the per unit water flow,
$\overline{h_s}$ = Per unit static head,
$\bar{h}$ = Per unit head at the turbine water admission.
$\overline{h_1}$ = Per unit head loss due to friction.
$\overline{T_w}$ = Water time constant or water starting time.
The block diagram for the above models is presented in Figure 4.

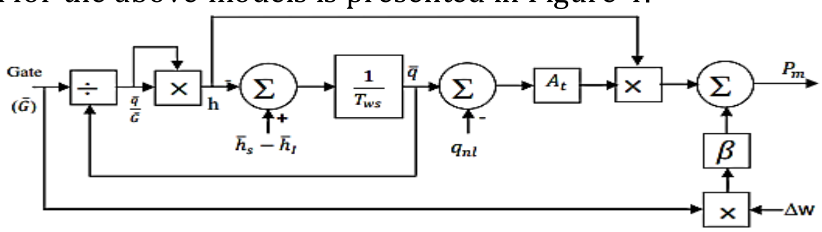

Figure 3: Block diagram of Penstock and turbine models

Hydro Turbine Input-Output Modeling Estimation

This section considered the equations describing the variation in flow rate, gate opening, runner blade movement of the hydro turbine and the mechanical power developed with respect to the turbine speed deviation and damping. Francis turbine is used in a wide range of application in the hydraulic industry because of its high efficiency of performance. As the developed power in the turbine varies with the flow rate, so does the system operate or attains a steady state when the flow through the p enstock gets constant. The equations governing the transient performance of the hydraulic turbines are based on the following assumptions:

i. Frictional resistance of the hydraulic turbine is neglected which implies that the blade is considered smooth.
ii. The water hammer on penstock is neglected
iii. The fluid is considered to be incompressible

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

iv. The velocity of water in penstock varies directly with gate opening
v. The developed power output of turbine is proportional to the product of head and velocity of flow.

$q = F(\text{gate}, \text{head})$

The flow rate through turbine in per unit system is given by equation below

$$\bar{q} = \bar{G}\sqrt{\bar{h}}$$

The turbine model is based on steady state measurements of the output power and water flow and the relation is given by equation below.

$$P_m = A_t h\,(q - q_{nL}) = \frac{1}{(\overline{G_{max}} - \overline{G_{min}})} \times h \times (q - q_{nL})$$

$$A_t = \frac{1}{(\overline{G_{max}} - \overline{G_{min}})}$$

Where:

P_m = Per unit mechanical power, q = Per unit water flow, A_t = Turbine gain, q_{nL} = Per unit no-load flow, $\overline{G_{max}}$ = Maximum full load per unit gate opening and
$\overline{G_{min}}$ = No-load per unit gate opening.

Practical turbine has less than 100% efficiency. It has a small speed deviation and damping effect due to the water flow in the turbine. Therefore, during a transient disturbance due to speed deviation, a modified form of equation (3.6) in terms of speed deviation is presented in equation below.

$$P_m = A_t h\,(q - q_{nL}) - \beta G \Delta\omega$$

$\Delta\omega$ = Speed deviation which defines the deviation of the actual turbine-generator speed from the normal speed.

$\beta G \Delta\omega$ = Speed deviation due to damping at the gate opening.

The hydraulic characteristics and mechanical power output of the turbine has been modeled here. The nonlinear characteristics of hydraulic turbine are neglected in this model. The complete MATLAB block diagram of hydro turbine model is shown in Figure 3.3. The actuator's (hydro-electric servo motor) output is the gate opening and it controls the valve to maintain a uniform speed by regulating the rate of water flow. The flow rate Q and net head H has been represented in equations (3.1)-(3.2). Here $(h_s - h - h_1)$ is entered as an input and the flow rate is the output signal. $h_s$has been assumed a static head with reference value of 1pu. Using a summation block, the signal $(h_s - h - h_1)$ is obtained. To find the actual water flow rate the no load flow is subtracted from Q using a sum block. Turbine frictional factor is neglected in this modeling. To get the value of mechanical power output Pm, equation 3.6 is used to establish the relation between the developed power at turbine, actual water flow rate and the Head.

Hydro turbine governor is a major part of hydro power plant. It is basically used for two purposes which are enumerated as follows:

i. Firstly, it develops mechanical power at the shaft of the generator which is fed to the synchronous generator for production of electricity.

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

ii. Secondly, it controls the variation of speed of the generator such that the generated frequency remains constant. The PID controller, hydroelectric servo system and hydraulic turbine are the main components of the hydro turbine governor. These block models of components are connected in such a manner that the generated frequency remains constant. The block diagram of hydro turbine governor is shown in Figure 3.4. The first element of the governor is PID controller. The error in speed and deviation of power is entered as input to the controller, which generate the position signal at the input of the hydro - electric servo motor. Furthermore, the servo motor responds by controlling the valve according to input signal to the servo mechanism. The valve is used to control the flow rate such that the generated frequency of the system remains constant.

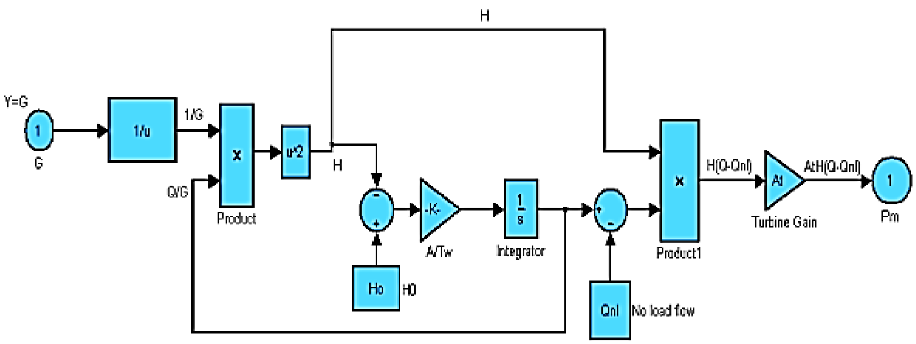

Figure 4: MATLAB/Simulink Block Diagram of Hydro Turbine Model.

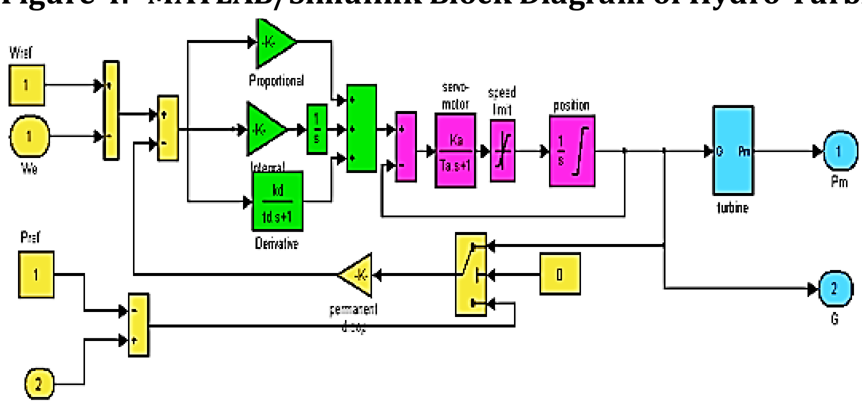

Figure 5. MATLAB/Simulink Model of Hydro Turbine Governor.

Mathematical modeling of a Synchronous Generator

This section presents the dynamic equations of three phase salient pole synchronous machine applied in the computer simulation. The electrical and electromechanical behaviour of most synchronous machines can be predicted from the dynamic equations that describe the three phase salient pole synchronous machine. The qdo voltage equations of a synchronous machine are clearly presented in equations below.

$$V_{qs} = r_s i_{qs} + P\lambda_{qs} + \omega_r \lambda_{ds}$$
$$V_{ds} = r_s i_{ds} + P\lambda_{ds} - \omega_r \lambda_{qs}$$
$$V_{os} = r_s i_{os} + P\lambda_{os}$$
$$V'_F = r'_F i'_F + P\lambda'_F$$
$$V'_{kd} = r'_{kd} i'_{kd} + P\lambda'_{kd}$$
$$V'_g = r'_g i'_g + P\lambda'_g$$
$$V'_{kq} = r'_{kq} i'_{kq} + P\lambda'_{Fq}$$

The flux linkages equations are presented in equations below.

$$\lambda_{qs} = L_q i_{qs} + L_{mq} i'_g + L_{mq} i'_{kq}$$
$$\lambda_{ds} = L_d i_d + L_{md} i'_F + L_{md} i'_{kd}$$
$$\lambda_o = L_{Ls} i_o$$
$$\lambda'_F = L_{md} i_{ds} + L_{md} i'_{kd} + L'_{FF} i'_F$$
$$\lambda'_{kd} = L_{md} i_{ds} + L_{md} i'_{kd} + L'_{kdkd} i'_{kd}$$
$$\lambda'_g = L_{mq} i_{qs} + L_{mq} i'_{kq} + L'_{gg} i'_g$$
$$\lambda'_{kq} = L_{mq} i_{qs} + L_{mq} i'_g + L'_{kqkq} i'_{kq}$$

The various leakage inductances and the mutual inductances are related by equations below.

$$L_q = L_{Ls} + L_{mq}$$
$$L_d = L_{Ls} + L_{md}$$
$$L'_{FF} = L_{LF} + L_{md}$$
$$L'_{kdkd} = L_{Lkd} + L_{md}$$
$$L'_{gg} = L_{Lg} + L_{mq}$$
$$L'_{kqkq} = L_{Lkq} + L_{mq}$$

Substituting equations gives rise to the modified flux linkage equations represented in below.

$$\lambda_{qs} = L_{Ls} i_{qs} + L_{mq}\left(i_{qs} + i'_g + i'_{kq}\right)$$
$$\lambda_{ds} = L_{Ls} i_{ds} + L_{md}\left(i_{ds} + i'_F + i'_{kd}\right)$$
$$\lambda'_F = L'_{LF} i'_F + L_{md}\left(i_{ds} + i'_F + i'_{kd}\right)$$
$$\lambda'_{kd} = L'_{Lkd} i'_{kd} + L_{md}\left(i_{ds} + i'_F + i'_{kd}\right)$$
$$\lambda'_g = L'_{Lg} i'_g + L_{mq}\left(i_{qs} + i'_g + i'_{kq}\right)$$
$$\lambda'_{kq} = L'_{Lkq} i'_{kq} + L_{mq}\left(i_{qs} + i'_g + i'_{kq}\right)$$

The electromechanical power output developed by the machine is given by equation below.

$$P_{em} = \frac{3}{2}\left(\omega_r\left(\lambda_d i_{qs} - \lambda_q i_{ds}\right)\right)$$

Where ω_r stands for the electrical speed which is related to the mechanical speed by equation below.

$$\omega_r = \left(\frac{P}{2}\right)\omega_m$$

$$P_{em} = \frac{3}{2} \times \frac{P}{2} \times \omega_m (\lambda_d i_{qs} - \lambda_q i_{ds})$$

For computer simulation purposes, the real and reactive power developed by the machine is given by equations below.

$$P = V_{qs} i_{qs} + V_{ds} i_{ds}$$
$$Q = V_{qs} i_{ds} - V_{ds} i_{qs}$$

The electromechanical torque developed is given by equation below.

$$T_{em} = \frac{3}{2} \times \frac{P}{2} (\lambda_d i_{qs} - \lambda_q i_{ds})$$
$$\frac{d\omega_{mr}}{dt} = \frac{1}{J} (T_{em} - T_{Load} - B\omega_{mr})$$
$$\theta_r = \int_0^t \omega_{mr} \quad dt$$

The above modeled equations for the hydro turbine plant and the synchronous generator were simulated in a MATLAB/SIMULINK environment.

Table 1 Feature of Kainji and Jebba Hydropower Reservoirs

Reservoir Features	Kainji	Jebba
First year of operation	1968	1984
Installed capacity (MW)	760	540
Design power plant factor	0.86	0.70
Number of generators	8	6
Reservoir flood storage capacity (Mm^3)	15,000	4,000
Reservoir flood level (m)	143.50	103.55
Maximum operating reservoir elevation (m.a.s.l)	141.83	103.00
Minimum operating reservoir elevation (m.a.s.l)	132.00	99.00
Maximum storage (active storage capacity), (Mm^3)	12,000	3,880
Minimum storage (Dead storage capacity), (Mm^3)	3,000	2,880

Source: Ifabiyi, I. P. (2021)

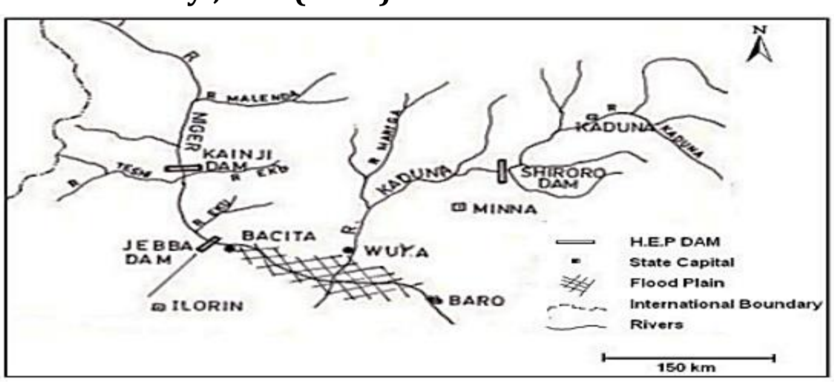

Figure 6: Location Map of the Kainji and Jebba Hydropower Reservoirs on Niger River
Source: Salami, A. W *et al* (2012)

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

Statistical Sampling of Hydro power reservoir variables for Kainji and Jebba Dam.

Total monthly historical data of Hydro Power reservoir variables for Kainji and Jebba stations were analyzed. The data include reservoir inflow (Mm^3), storage (Mm^3), reservoir elevation (m), turbine release (Mm^3), net generating head (m), plant use coefficient, tail race level (m), evaporation losses (Mm3) and energy generation (MWh). These data were collected from Power Holding Company of Nigeria (PHCN) for a period of forty two years (1980-2021) for Kainji and twenty eight years (1994-2021) for Jebba station. The summary of the statistical analysis is presented in Tables 3.2 and 3.3 for Kainji and Jebba Hydro Power reservoirs.

$$\text{Mean} = \bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \text{S.D} = \sqrt{\frac{\sum fd^2}{\sum f}} = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

$$\text{Spearman coefficient of variation} = \text{CV} = 1 - \frac{6\sum d_i{}^2}{N(N-1)}$$

Where: d_i = Maximum − minimum value and N = number of years in review.

Table 2: Summary of Statistical Analysis of the Kainji Hydro Power Data (1980 - 2021)

Reservoir Elements	Mean	Median	S.D	C.V	Minimum	Maximum	Skew
Reservoir Inflow (Mm^3)	2504.35	2408.77	632.93	0.25	1396.93	3653.04	0.44
Reservoir Storage (Mm^3)	8058.58	8088.18	722.04	0.09	6654.53	9258.92	-0.17
Reservoir Elevation (m)	138.17	138.12	0.67	0.00	136.86	139.34	0.00
Turbine Release (Mm^3)	1881.96	1880.54	419.47	0.22	1131.05	2806.00	0.18
Net Generating Head (m)	38.57	38.48	0.59	0.02	37.63	39.90	0.41
Energy Generation (MWh)	178570.26	158016.25	73090.93	0.41	81553.53	378612.85	1.13
Plant Use Coefficient	0.39	0.35	0.15	0.39	0.17	0.80	1.06
Tail Race Level (m)	100.97	100.37	1.92	0.02	98.02	103.83	0.26
Evaporation Loss (Mm^3)	145.20	146.00	5.30	0.04	138.52	152.10	-0.09

Note: S.D = Standard deviation, C.V = Coefficient of variation

Table 3 Summary of Statistical Analysis of the Jebba Hydro Power Data (1994 - 2021)

Reservoir Elements	Mean	Median	S.D	C.V	Minimum	Maximum	Skew
Reservoir Inflow (Mm^3)	2647.90	2474.85	729.23	0.28	1588.86	4154.26	0.51
Reservoir Storage (Mm^3)	3595.74	3594.00	79.81	0.02	3423.55	3751.58	0.13
Reservoir Elevation (m)	102.01	101.99	0.34	0.00	101.22	102.60	-0.17
Turbine Release (Mm^3)	2182.05	2186.21	297.70	0.14	1566.66	2572.62	-0.37
Net Generating Head (m)	28.15	28.22	0.54	0.02	26.61	29.07	-1.05
Energy Generation (MWh)	175134.17	163525.08	54346.26	0.31	103542.36	282039.95	0.77
Plant Use Coefficient	0.47	0.45	0.15	0.33	0.26	0.72	0.20
Tail Race Level (m)	73.69	73.61	0.40	0.01	73.09	74.54	0.64
Evaporation Loss (Mm^3)	49.28	49.30	1.90	0.04	45.57	51.27	-0.77

Note: S.D = Standard deviation, C.V = Coefficient of variation

Source: Power Holding Company of Nigeria (PHCN), 2012

Artificial Neural Network (ANN) Data Analysis of Kainji and Jebba HP Station

The data sets were partitioned into three; training, testing and holdout samples for each of the stations. The training data records were used to train the neural network in order to obtain a model. The testing sample is an independent set of data records used to track errors during training in order to prevent

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

overtraining. The holdout sample is another independent set of data records used to assess the final neural network; the error for the holdout sample gives an estimate of the predictive ability of the model. In this study, multilayer perceptron (MLP) was adopted with sigmoid activation function both at hidden and output layers. A normalized method of rescaling was used for scaling the Hydro Power reservoir variables.

Results and Analysis

The ANN generated network structure is presented in Figure 4.1 while a plot of predicted value against the energy generated for Kainji Hydro power is represented in Figure 4.2 respectively.

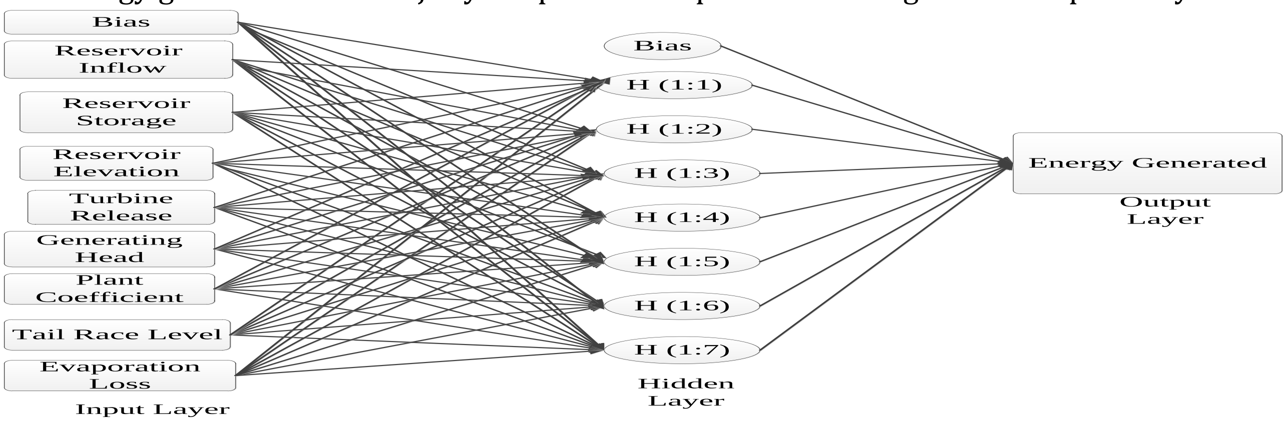

Figure 7: Generated Network Structure

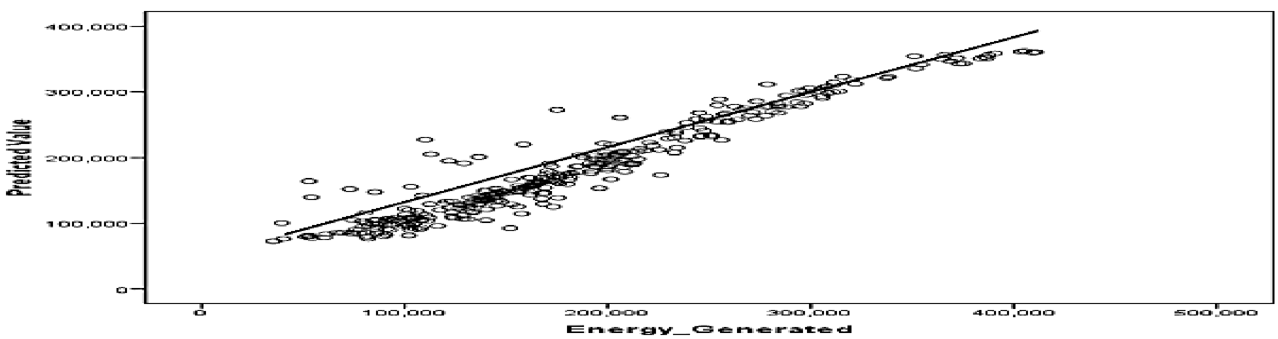

Figure 8: Scatter Plot of Predicted and Observed Energy Generated for Kainji HP

Similarly, the scatter plot of the predicted value against the Energy generated for Jebba hydro power plant is presented in Figure 9

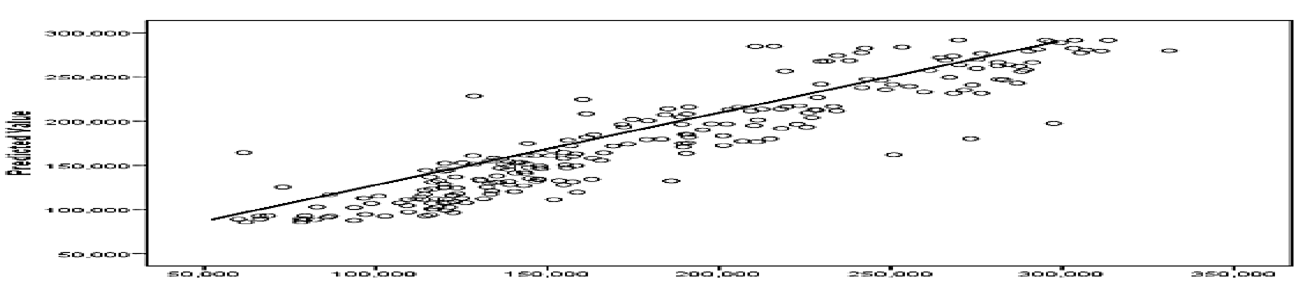

Figure 9: Scatter Plot of Predicted and Observed Energy Generated for Jebba HP.

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

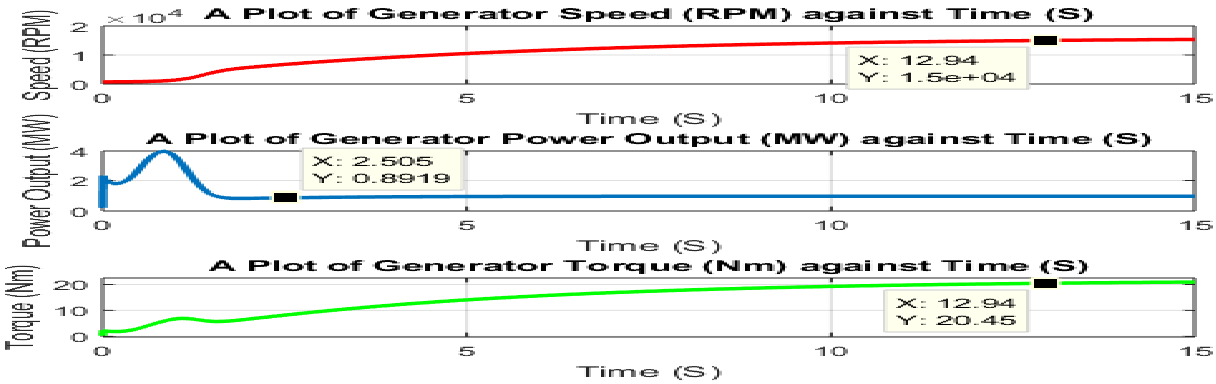

Figure 10. A Plot of Generator Speed, Power Output & Torque against Time

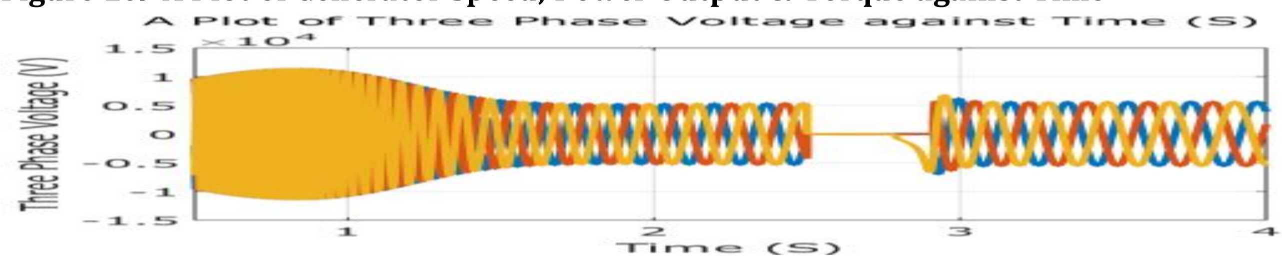

Figure 11: A Plot of Three Phase Voltage against Time under 3LG-Fault

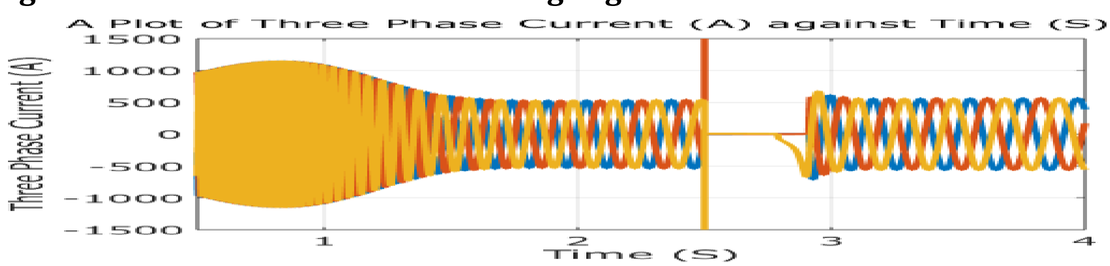

Figure 12. A Plot of Three Phase Current against Time under 3LG-Fault

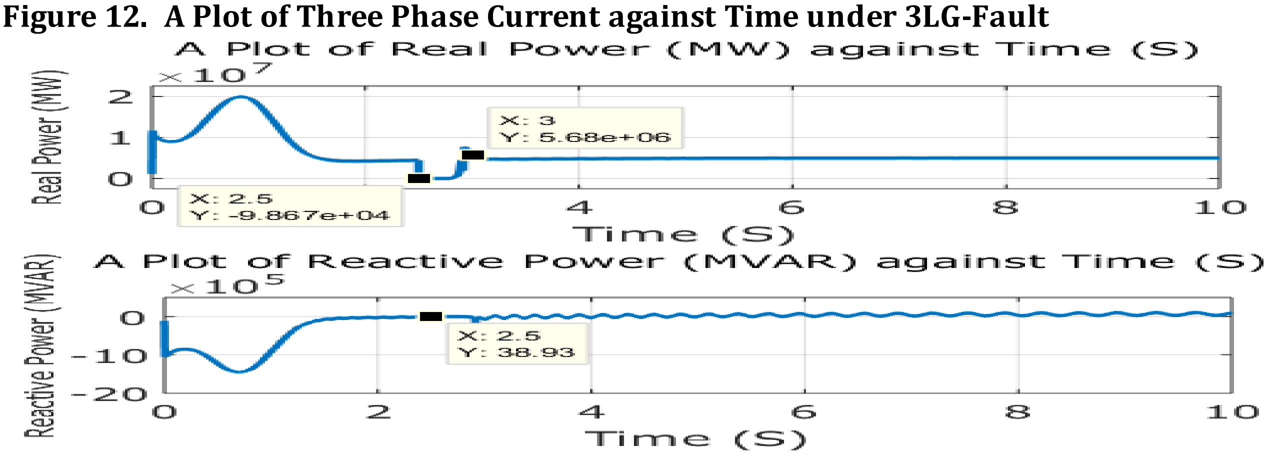

Figure 13. A Plot of Generator Real & Reactive Power against Time under 3LG-Fault

Science and Engineering Research Journal
ISSN: 2997-6227 |
Volume 13 Issue 3, July-September, 2025
Journal Homepage: https://ethanpublication.com/articles/index.php/E5
Official Journal of Ethan Publication

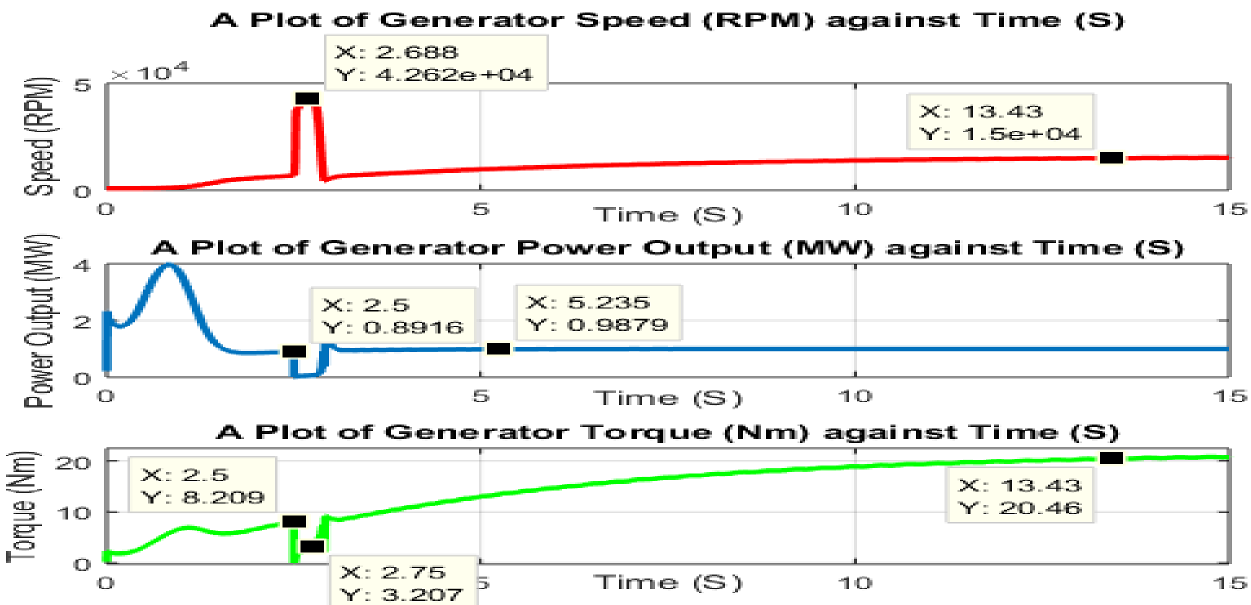

Figure 14 Plot of Generator Speed, Power Output & Torque against Time under 3LG-Fault

Discussion of Findings and Summary

The findings of this study also suggest that the synchronous generator is capable of producing a stable three-phase voltage, which is essential for power generation and transmission. This is consistent with the study of Bhattarai *et al*. (2019), who employed novel trends in modeling techniques of Pelton Turbine bucket for increased renewable energy production. Their study highlighted the importance of optimizing turbine performance to enhance energy generation, which is also relevant to the operation of synchronous generators. Furthermore, the study by Kokubu *et al*. (2012) on the performance improvement of a micro eco cross-flow hydro turbine also demonstrated the importance of optimizing turbine design parameters to enhance energy generation. This is consistent with the findings of this study, which suggest that the synchronous generator is capable of producing a stable three-phase voltage. Kainji and Jebba hydropower reservoir variables were modeled for energy generation using neural network multilayer perceptron with sigmoid function as activator. Neural network model yielded a good forecast for Kainji and Jebba hydropower reservoirs with correlation coefficients of 0.89 and 0.77 respectively. These values showed strong linear relationship between the observed and predicted energy generation and this is an indication that the Artificial Neural Network model is reliable for modeling of hydropower reservoirs for energy generation. The dynamic characteristics of the turbine and the synchronous generator were obtained through mathematical modeling and simulation processes. The results obtained showed that the turbine and the generator performance are affected by parameter variation such as the governor settings and controller adjustments.

Conclusion

The general non-linear mathematical models of hydraulic turbine have been presented in this thesis. This model is suitable for dynamic studies of hydro power plant. At the same time, it also focused on how to implement the developed mathematical models into a simulation process to showcase the physical characteristic performance of the composite system. Severe disturbances were examined on dynamic model of the power plant and power systems. Results showed sufficient accuracy of the model for the whole working range. The current turbine models with minor refinements will improve accuracy over the entire operating range. Kainji and Jebba hydropower reservoir variables were modeled for energy generation using neural network multilayer perceptron with sigmoid function as activator. Neural network model yielded a good forecast for Kainji and Jebba hydropower reservoirs with correlation coefficients of 0.89 and 0.77 respectively. These values showed strong linear relationship between the observed and predicted energy generation and this is an indication that the NN model is reliable for modeling of hydropower reservoirs for an efficient energy generation.

Contributions to Knowledge

This work serves as a useful guide for power engineering experts and research students who want to use an appropriate hydro power plant model including the penstock, turbine, governor, and generator for stability studies. This thesis provides required background knowledge for studying the effect of actual data on the model parameters and behavior of a hydro turbine performance method. The review and comparison of different models help the electrical engineers to choose the best model for specific studies. Furthermore, the appropriate tuning of the conventional controller and governor using a simple approach has been presented which is very useful for educational purposes and real time applications for efficient power generation.

References

Aatif, S., & Arbab, M. N. (2015). Capacity estimation of power generation from MSW of Peshawar City. *International Journal of Computer Applications*, *111*(15).

Bahrami, S., Tribes, C., Devals, C., Vu, T. C., &Guibault, F. (2016). Multi-fidelity shape optimization of hydraulic turbine runner blades using a multi-objective mesh adaptive direct search algorithm. Applied Mathematics and Modelling, 40, 1650–1668.

Bhattarai, S., Vichare, P., Dahal, K., Al Makky, A., &Olabi, A. G. (2019). Novel trends in modelling techniques of Pelton Turbine bucket for increased renewable energy production. Renewable and Sustainable Energy Reviews, 112, 87-101.

Bordin, C.; Skjelbred, H.I.; Kong, J.; Yang, Z. Machine learning for hydropower scheduling: State of the art and future research directions. *ProcediaComput. Sci.* 2020, *176*, 1659–1668 dos Santos Sousa, R., Dewes, J. J., Rauch, H. P., Sutili, F. J., & Hörbinger, S. (2025). First results of soil and water bioengineering interventions to stabilise and control erosion processes in hydroelectric power plant reservoirs in Brazil. *Ecological Engineering*, *211*, 107458.

Ermenc, J. J. (2019). Small Hydraulic Prime Movers for Rural Areas of Developing Countries: A Look at the Past. In *Renewable Energy Resources and Rural Applications in the Developing World* (pp. 89-113). Routledge.

Evans, P. S. (2017, January). Water turbines of the woods point goldfield 1866–1867. In *19th Australasian engineering heritage conference: Putting water to work: Steam power, river navigation and water supply* (pp. 76-89). Engineers Australia Mildura, VIC.

Kokubu, K., Kanemoto, T., Son, S. W., & Choi, Y. D. (2012). Performance improvement of a micro eco cross-flow hydro turbine. Journal of Advanced Marine Engineering and Technology (JAMET), 36(7), 902-909.

Lee, J. H., Kim, T. S., & Kim, E. H. (2017). Prediction of power generation capacity of a gas turbine combined cycle cogeneration plant. *Energy*, *124*, 187-197.

Lewis, B. J., Cimbala, J. M., & Wouden, A. M. (2014, March). Major historical developments in the design of water wheels and Francis hydroturbines. In *IOP Conference Series: Earth and Environmental Science* (Vol. 22, No. 1, p. 012020). IOP Publishing.

Liu, J., Zhang, J., Huang, Z., Xu, H., Bai, Y., & Wang, G. (2025). Modeling the hydraulic roughness of a movable flatbed in a sand channel while considering the effects of water temperature. *International Journal of Sediment Research*, *40*(1), 31-44.

Naderi, S., Banifateme, M., Pourali, O., Behbahaninia, A., MacGill, I., & Pignatta, G. (2020). Accurate capacity factor calculation of waste-to-energy power plants based on availability analysis and design/off-design performance. *Journal of Cleaner Production*, *275*, 123167.

Osman, A. I., Fang, B., Zhang, Y., Liu, Y., Yu, J., Farghali, M., & Yap, P. S. (2024). Life cycle assessment and techno-economic analysis of sustainable bioenergy production: a review. *Environmental Chemistry Letters*, *22*(3), 1115-1154.

Sajjad, W. (2025). Capacity-Specific Life Cycle Assessment of Pakistani Coal-Fired Power Plants: Energy Flows, Environmental Impacts, and Renewable Integration Routes. doi: 10.20944/preprints202502.2171.v1

Salami, A. W. and Sule, B. F. (2012), Optimal Water Management Modelling for Hydropower System on River Niger in Nigeria, International Journal of Engineering, FASCICULE 1 (ISSN 1584-2665), Annals of Faculty of Engineering Hunedoara, Tome X, pp. 185-192.

Soares, L. M. V., & do CarmoCalijuri, M. (2022). Restoration from eutrophication in interconnected reservoirs: Using a model approach to assess the propagation of water quality improvements downstream along a cascade system. *Environmental Modelling & Software*, *149*, 105308.

Suhaime, M. A., Tokit, E. M., Sa'at, F. A. Z. M., Rosli, M. A. M., Herawan, S. G., Oktarina, R., & Setyandito, O. (2023, December). Preliminary theoretical design of crossflow water turbine for rainwater rooftop harvesting system. In *IOP Conference Series: Earth and Environmental Science* (Vol. 1281, No. 1, p. 012024). IOP Publishing.

Turgeon, K.; Trottier, G.; Turpin, C.; Bulle, C.; Margni, M. Empirical characterization factors to be used in LCA and assessing the effects of hydropower on fish richness. *Ecol. Indic.* 2021, *121*, 107047