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ANALYZING PIECEWISE LINEAR ECONOMIC-MATHEMATICAL MODELS CONSIDERING UNACCOUNTED FACTORS: A STUDY IN 3-DIMENSIONAL VECTOR SPACE

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Abstract:
This paper builds upon the foundation established in prior publications [1-5, 12], which introduced the theory of constructing piecewise-linear economic mathematical models within finite-dimensional vector spaces, accounting for the impact of unaccounted factors. These models offer a promising framework for predicting and managing economic processes under conditions of uncertainty, providing a means to define economic process control functions within multidimensional vector spaces. However, it is essential to acknowledge the inherent challenges in addressing uncertainty within economic processes. The lack of a precise definition for "uncertainty" in economic contexts, incomplete classifications of its manifestations, and the absence of a clear mathematical representation contribute to the complexity of solving predictive and control problems. The economic landscape is characterized by multidimensionality and spatial heterogeneity, compounded by the temporal variability of multifactor economic indicators and their changing rates.
This paper navigates these complexities and uncertainties, offering insights and methodologies to elevate the solution of economic process prediction and control problems to a higher level of sophistication.

Keywords: Piecewise-linear models, economic mathematical models, uncertainty, economic processes, multidimensionality.

I. Introduction. Formulation of the problem

In publications [1-5, 12] theory of construction of piecewise-linear economic mathematical models with regard to unaccounted factors influence in finite-dimensional vector space was developed. A method for predicting economic process and controlling it at uncertainty conditions, and a way for defining the economic process control function in m-dimensional vector space, were suggested.

In addition to this we should note that no availability of precise definition of the notion "uncertainty" in economic processes, incomplete classification of display of this phenomenon, and also no availability of its precise and clear mathematical representation places the finding of the solution of problems of prediction of economic process and this control to the higher level by its complexity. Many-dimensionality and spatial in homogeneity of the occurring economic process, time changeability of multifactor economic indices and also their change velocity give additional complexity and uncertainty. Another complexity of the problem is connected with reliable construction of such a predicting vector equation in the consequent small volume $\Delta V_{n+1}(x_1, x_2, \ldots x_m)$ of finite-dimensional vectorspace that could sufficiently reflect the state of economic process in the subsequent step. In other words, now by means of the given statistical points (vectors) describing certain economic process in the preceding volume N
$V = \sum \Delta V_N (x_1, x_2, \ldots x_m)$ of finite-dimensional vector space R_m one can construct a predicting vector equation $N+1$

$Z_{n+1}(x_1,x_2,...x_m)$ in the subsequent small volume $\Delta V_{n+1}(x_1,x_2,...x_m)$ of finite-dimensional vector space. The goal of our investigation is to formulate the notion of uncertainty for one class of economical processes and also to find mathematical representation of the predicting function $Z_{n+1}(x_1,x_2,...x_m)$ for the given class of processes depending on so-called unaccounted factors functions. In connection with what has been said, below we suggest a method for
constructing a predicting vector equation $Z_{n+1}(x_1,x_2,...x_m)$ in the subsequent small volume $\Delta V_{n+1}(x_1,x_2,...x_m)$ of finite-dimensional vector space [1-7, 14].

II. Materials and methods:

In these publications, the postulate spatial-time certainty of economic process at uncertainty conditions in finite-dimensional vector space" was suggested, the notion of piecewise-linear homogeneity of the occurring economic process at uncertainty conditions was introduced, and also a so called. The unaccounted parameters N
influence function $\omega_n(\lambda_n^{k_n},\alpha_{n-1,n})$ influencing on the preceding volume $V = \sum \Delta V_n$ of economic process was $N = 1$ suggested.

$$z_n = z_1\left\{1 + A\left[1 + \omega_n(\lambda_n,\alpha_{n-1,n}) + \sum_{i=2}^{n-1}\omega_i(\lambda_i^{k_i},\alpha_{i-1,i})\right]\right\} \quad (1)$$

Here

$\omega_i(\lambda_i^{k_i},\alpha_{i-1,i}) = \lambda_i^{k_i}\cos\alpha_{i-1,i}$

i ki-1 ki-1 k1 ik zi-1 - zi-1 ai 1 - zi-1 z1(z1 -a1) k1 ki-1 k1 cos i-1,i (2)

1 - 1z1(zi-1 - zi-1)a2 -a1z1 -a 1

ki-2 k-1 ki-1 (ai -zi-2)(ai) ki-1

μi=(μi-1 -μi-1) (a i 1 -z ik-1 i-1) 2 , for i-1≥ i-1 (3)

$\omega_n(\lambda_n,\alpha_{n-1,n}) = \lambda_n\cos\alpha_{n-1,n}$

z n-1 - z nk-n1-1 a n 1 - z n-k n-11 z 1(z 1k1 -a 1)

n

k1 kn-1 k1 cos n-1,n (3.1)

1 - 1z1(zn-1 - zn-1) a2 - a1z1 -a 1

k1

a2 -a1 z1 -a 1

k1

$A = (\mu_1 - \mu_1)$ k1 (4)

z1(z1 -a1)

kn-2 k 1 kn-1 (an -zn-2)(an 1 - z_{n-1}^{n-}) k

μn=(μn-1 -μn-1) kn-1 2 , n-1 ≥ μ_{n-1}^{n-1} (5)

(an 1 -zn-1)

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On this basis, it was suggested the dependence of the n-th piecewise-linear function z_n on the first piecewise-linear
 ^{kn} function z_1 and all spatial type unaccounted parameters influence function $_n$ ($_n$,$_{n-1,n}$) influencing on the preceding interval of economic process, in the form Eqs. (1)–(5):

 n-1 2

$z_n = z_1 \{1 + A| 1 + \omega_n(\lambda_n, \alpha_{n-1,n}) + \sum \omega_i(\lambda_i^{k_i}, \alpha_{i-1,i})$ (6)

i=2

Where $_i$ ($_{iki}$,$_{i-1,i}$) iki
$cos_{i-1,i}$

i ki-1 ki-1 $k1$
ik zi-1 - zi-1 ai1 - zi-1 $z1(z1 -a1)$
k1 $_k i$-1 $k1$ $cos_{i-1,i}$ (7)
1 -1$z1(zi$-1 - zi-1)$a2 -a1z1 -a$ 1

are unaccounted parameters influence functions influencing on the preceding V_1, V_2,..V_i small volumes of economic process;

ki-2 ki-1 ki-1 (ai -zi-2)(ai1 -zi-1) k
i (i-1 -i-1) ki-1 2 , for μi-$_{1\geq}$ μ$_{i-1}^{i-1}$ (8)

(ai1 -zi-1)

are arbitrary parameters referred to the i-th piecewise-linear straight line. And the parameters $_i$ are connected with the parameter $_{i-1}$ referred to the (i-1)-th piecewise-linear straight line, in the form Eq. (8);

$_{k1}$
$a2 -a1$ $z1 -a$ 1
k_1
A ($_1$ -$_1$) $_k$1 (9)
$z1(z1 -a1)$
is a constant quantity;

n (n ,n-1,n) $ncos$$n$-1,$n$

$_n$ zn-1 - znk-n1-1 an1 - zn-kn-11 z1(z1$k1$ -a1)
k1 $_k n$-1 $k1$ $cos_{n-1,n}$ (10)
1 -1$z1(zn$-1 - zn-1) $a2$ - $a1z1 -a$ 1

is the expression of the unaccounted parameters influence function that influences on subsequent small volume V_N of finite-dimensional vector space. And the parameter $_n$ referred to the n- piecewise-linear straight line is of the form:

kn-2 k 1
μn=(μn-1 -μn-1) kn-1 2 n-1≥n-1 (11) (a -z)(a_1 - z_{n-1}^{n-})
, kn-1 n n-2
(an1 -zn-1) $n$$kn$-1

Here the parameter $_n$ is connected with the parameter $_{n1}$ of the preceding (n-1)-th piecewise-linear vector equation of the straightline in the form Eq. (11). Thus, in finite-dimensional vector space, the system of statistical points (vectors) is represented in the vector form in the form of N piecewise-linear straight lines depending on the vector function of the first piecewise-linear straight-line z_1 a_1 a_2, and also on the unaccounted parameters influence function $_n$ ($_n$,$_{n\text{-}1,n}$) in all the investigated preceding volume of finite-dimensional vector space R_m.

After that, in publications [6-11,13-15] a solution was found of solve a problem on prediction of economic process and its control at uncertainty conditions in finite-dimensional vector space. It became clear, that the unaccounted parameters influence functions □*n* (□*n*,□*n*-1,n) are integral characteristics of influencing external factors occurring in environment that are not a priori situated in functional chain of sequence of the structured model but render very strong functional influence both on the function and on the results of prediction quantities Eq. (6). It is impossible to fix such a cause by statistical means. This means that the investigated this or other economic process in finite dimensional vector space directly or obliquely is connected with many dimensionality and spatial inhomogenlity of the occurring economic process, with time changeability of multifactor economic indices, vector and their change velocity. This in its turn is connected with the fact that the used statistical data of economic process in finitedimensional vector space are of inhomogeneous in coordinates and time unstationary events character.

We assume the given unaccounted factors functions □*n* (□*n*,□*n*-1,n) hold on all the preceding interval of finite-dimensional vector space, the uncertainty character of this class of economic process. In such a statement, the problem on prediction of economic event on the subsequent small volume □$V_{N□1}$ of finite-dimensional vector space will be directly connected in the first turn with the enumerated invisible external facts fixed on the earlier stages and their combinations, i.e., the functions □*n* (□*n*,□*n*-1,n) that earlier hold in the preceding small volumes □V_1,□V_2,.....□V_N of finite-dimensional vector space. Therefore, by studying the problem on prediction of economic process on subsequent small volume □$V_{N□1}$ it is necessary to be ready to possible influence of such factors. In connection with such a statement of the problem, let's investigate behavior of economic process in subsequent small volume □$V_{N□1}$ finite-dimensional vector space under the action of the desired unaccounted parameters function □*n* (□*n*,□*n*-1,n) that was earlier fixed by us in preceding small volumes □V_n of finite-dimensional vector space, i.e.,

□2 (□2 ,□1,2) , □3 (□3 ,□2,3) , …., □*N* (□*N* ,□N-1, *N*) . In connection with what has been said, the problem on prediction of economic process and its control in finite-dimensional vector space may be solved by means of the introduced unaccounted parameters influence function □*n* (□*n* ,□*n*-1,*n*) in the following way. Construct the (N+1)-the

□ □*kN* □ □□*kN*

vector equation of piecewise-linear straight line $z_{N□1}$=$^z{}_N$+μN□1($a_{N□2}$□ $^z{}_N$) depending on the vector equation of □the first piecewise-linear straight line $^z{}_1$ and the desired unaccounted parameter influence function □□(□□,□□-1,□)

That we have seen in one of the preceding small volumes □V_1,□V_2,.....□V_N of finite-dimensional vector space. For that in Eqs. (6)–(11) we change the index n by (*N*□1) and get:

□ □□□ □ ∑*N* *ki* □□

zN □1=*z*1□1+*A*□1+ *ωi* (*λi* ,*αi*-1,*i*)□*ωN* □1(*λN*□1,*αN*,*N*□1)□□ (12)

□ □ *i*=2 □□

Here

□*i* (□*iki* ,□*i*-1,*i*) □□*iki cos*□*i*-1,*i* □

ki □ □*ki*-1 □ □*ki*- 1 □□□*k*1 □

□*i* *zi*-1 - *zi*-1 *ai*□1 - *zi*-1 *z*1(*z*1 -*a*1)

□ *k*1 □ □□□ □*kil*-1 □ □□□□*k*1 □*cos*□ *i*-1,*i* (13)

□1 -□1*z*1(*zi*-1 - *zi*-1)*a*2 -*a*1*z*1 -*a* 1

□□□*ki*-2 □□□□*ki*-1 *ki*-1 (*ai* -*zi*-2)(*ai*□1 -*zi*-1) *k*

μ*i*=(μ*i*-1 -μ*i*-1) □ □*ki*-1 2 , □*i*-1 ≥ μ_{i-1}^{i-1} (14)

(*ai*□1 -*zi*-1)

□□□□□$_{k1}$ □□^{k1} *a*2 -*a*1 *z*1 -*a*1

A (1 -1) k1 (15) z1(z1 -a1)
N1(N1,N,N1) N1cosN,N1

N1 zN - zNkN aN2 - zNkN z1(z1k1 -a1)
 k1   kN k1 cos N,N1 (16)
1 1 z1(zN - zN) a2 -a1 z1 -a1
 k   kN zN-1 N-1)(
kN (aN1 - aN2 -zN) kN
μN1=(μN -μN) kN 2 ,N≥N (17)
(aN2 -zN)

For the behavior of economic process on the subsequent small volume V_{N1} of finite-dimensional vector space to be as in one of the desired preceding ones in small volume ΔV_β it is necessary that the vector equations of   piecewise-linear straight lines z_{N1} and z_β to be situated in one of the planes of these vectors and to be parallel to one another, i.e.

$z_{N1} = Cz_\beta$ (18)

In connection with what has been said, to V_{N1} finite-dimensional space there should be chosen such a vector-point
    kN   kβ1
a_{N2} that the piecewise-linear straight lines z_{N1} (a_{N2} z_N) and z_β ($a_{\beta1}$ $z_{\beta1}$) could be situated in the same plane of these vectors and at the same time be parallel to each other (Fig. 1).

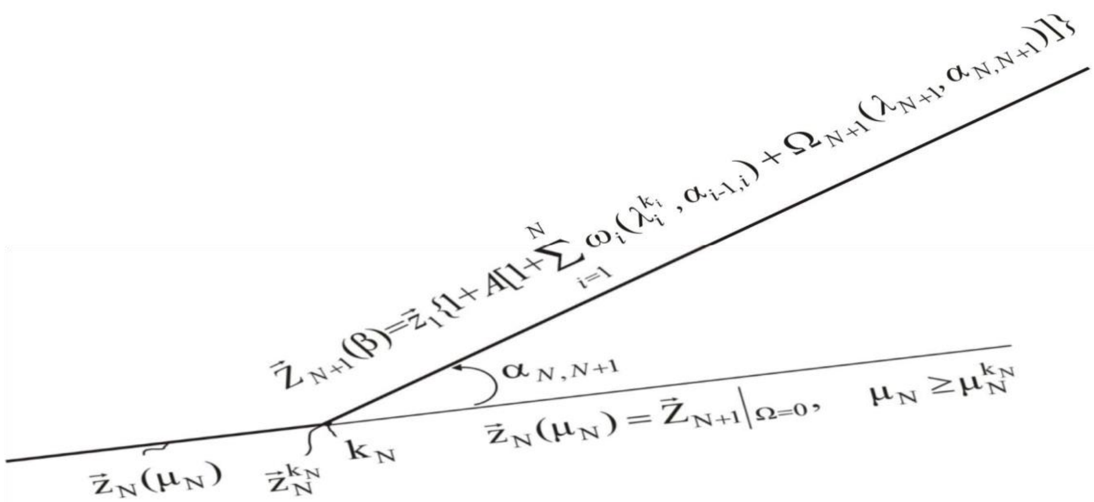


Fig. 1. The scheme of construction of prediction function of economic process $\vec{Z}_{N1}()$ at uncertainty conditions in Finite-dimensional vector space R_m. Prediction function $Z_{N1}()$ will lie in the same plane with one of the desired preceding -the piecewise-linear straight line and will be parallel to it.

In other words, they should satisfy the following parallelism condition:
 kN kβ1
$(a_{N2} - z_N) = C(a_{\beta1} - z_{\beta1})$ (19)

Here
 M M

aN□2 □ □aN□2,mim , aβ□1 □ □aβ□1,m im ,
m□1 m□1
□kN M kN □□□kβ□1 M kβ□1 □
zN □ □zN,mim , zβ□1 □ □ƶβ□1,m im
m□1 m□1
Excluding in Eq. (19) the parameter C, we get:
aN□2,1 □ zNkN,1 aN□2,2 □ zNkN,2 a N□2,M □ zkN,N M = =.....=
(20)

kβ□1 kβ□1 kβ□1
aβ□1,1 □ zβ□1,1 aβ□1,2 □ zβ□1,2 aβ□1,M □ z β□1,M
□□It is easy to define from system Eq. (20) the coefficients of the vector
aN□2 :
kβ□1 kN aβ□1,2 □ z β□1,2 kN
a N□2,2 □ zN,2 □ kβ□1 (aN□2,1 □ zN,1)

aβ□1,1 □ zβ□1 , 1
kβ□1 kN aβ□1,3 □ z β□1,3 kN
a N□2,3 □ zN,3 □ (a N□2,1 □ zN,1) aβ□1,1 □
zβ□1,1

k
a □ z β□1
a N□2,M □ zkN,NM □ β□1,M β□1,M (a □ z kN,NM□1) (21)

aβ□1,M□1 □ zβk□β□11,M□1 N□2,M□1
□□In this case, the vector aN□2 will have the following final form:
□ □ □ □ □
aN□2 = aN□2,1i1 □ aN□2,2i2 □ aN□2,3i3 □......□ aN□2,M iM (22)

□
As the coordinates of the point (of the vector) aN□2 now are determined by means of the piecewise-linear
□ □ □□□□k β□1
vector zβ □ aβ□1 □ zβ□1 taken from one of the preceding stage of economic process, it is appropriate to denote them
□ □
in the form aN □2 (□) [8-13]. This will show that the coordinates of the point aN□2 (3) were determined by means of □
piecewise-linear straight line zβ. In this case it is appropriate to represent Eq. (22) in the following compact form:
□ M □
aN□2 (□) □ □aN□2,m(□)im (23)
m□1

□ □

Now, in the system of Eqs. (12)–(17), instead of the vector $a_{N □1}$ we substitute the value of the vector $a_{N □2}$ (□) , and also instead of □$_{N□1}$(□$_{N□1}$,□$_{N,N□1}$) introduce the denotation of the so-called predicting influence function with regard to unaccounted parameters □$_{N□1}$(□$_{N□1}$,□$_{N,N□1}$) . In this case the prediction function of the economic process □

$Z_{N□1}$(□) with regard to influence of prediction function of unaccounted parameters □ $_{N □1}$($\lambda_{N □1}$,$\alpha_{N,N □1}$) will take the following form:

□ □□□ □ N □□

$(\beta)=z_1\{1+A| 1+ \sum\omega_i(\lambda_i^{k_i}, \alpha_{i-1,i})$

Z N□1 □□N □1(λN□1,$\alpha N,N$ □1)□□ (24)

□ □ i=2 □□

Here

□i (□ki i ,□i-1,i) □ □iki cos□i-1,i □

iki z□i-1 - z□ik-1i-1 a□i□1 - z□□i-k1i-1 z□1(z□1k1 -a□1)

□

□ $_k$1 □ □□□ □$_k$$i$-1 □ □□□□□$_k$1 □$cos$□ $_{i-1,i}$ (25)

□1 -□1z1(zi-1 - zi-1)a2 -a1 z1 -a 1

□□□ki-2 □ □□k -1

(a -z)(a) ki-1 i i-2 i□ k

$1 \geq \mu_{i-1}^{i-1}$

μi=(μi-1 -μi-1) □ □ki-1 2 , □i- (26)

(ai□1 -zi-1)

□□□□□□^{k1} □

a2 -a1 z1 -a 1

A □(□1k1 -□1) □□□□k1 □ (27)

z1(z1 -a1)

And the prediction function of influence of unaccounted parameters □$_{N□1}$(□$_{N□1}$,□$_{N,N□1}$) will take the form:

□N□1(□N□1,□N,N□1) □□N□1cos□N,N□1 (28)

□ □kN □ □□kN □□□k1 □

□N□1 zN - zN aN□2 (□)- zNz1(z1 -a1)

□$_N$□$_1$ □ $_k$1 □ □□□ □$_k$N □ □□□□k1 □ (29)

□1 □□1 z1(zN - zN) a2 -a1z1 -a1

□ □kN-1 □ □□kN

kN (aN□1 -zN-1)(aN□2(□)-zN) kN

μN□1=(μN -μN) □ □kN 2 ,□N≥□N (30)

(aN□2 (□)-zN)

□

Here the vector $a_{N□2}$ (□) is determined by Eq. (23).

Note the following points. It is seen from Eq. (11) that for □$_N$ □ □$_N$$^{k\,N}$ the value of the parameter □$_{N □1}$ □ 0 . By this fact from Eq. (28) it will follow that the value of the predicting function of influence of unaccounted parameters □N□1(□N□1,□N ,N□1) will equal:

□N□1(□N□1,□N ,N□1) =0 for □N □1 □ 0

□N□1(□N□1,□N , N□1) □ 0 for □N □1 □ 0 (31)

This will mean that the initial point from which the (N+1)-th vector equation of the prediction function of economic
process $Z_{N+1}(\lambda)$ will enanimate, will coincide with the final point of the n-th vector equation of piecewise-linear
straight line z_N and equal:

$$Z_{N+1} = z_1 + A \sum_{i=2}^{N} k_i \omega_i (\lambda_i, \alpha_{i-1,i}), \text{ for } \mu_{N+1} = 0 \qquad (32)$$

For any other values of the parameter $\mu_{N+1} \neq 0$ the points of the $(N+1)$ -th vector equation will be determined by
Eq. (24). It is seen from Eq. (28) that $\mu_{N+1} = 0$ and $\omega_{N+1}(\mu_{N+1} = 0; \alpha_{N,N+1}) = 0$ will follow $cos\alpha_{N,N+1} = 0$ and
$\lambda_{N+1} = 0$. This will correspond to the case when the influence of external unaccounted factors on subsequent small volume ΔV_{N+1} are as in the preceding small volume ΔV_N of finite-dimensional vector space. In this case it suffices to
continue the preceding vector equation z_N to the desired point $\lambda_{N+1} = \lambda^*_{N+1} = k_N$ of subsequent small volume of finite-dimensional vector space.

The value of the vector function $Z_{N+1}(\lambda^*_{N+1}) = z_N(\lambda^*_{N+1}; \lambda_N, \alpha_{N+1,N})$ at the point $\lambda_{N+1} = \lambda^*_{N+1}$ will be one of the desired prediction values of economic process in subsequent small volume ΔV_{N+1}. In this case, the value of the controlled parameter of unaccounted factors will be equal to zero, i.e.,
$\omega_{N+1}(\mu_{N+1} = 0; \lambda_{N+1} = 0; \cos\alpha_{N,N+1} = 0; \alpha_{N,N+1} = 0) = 0$
For any other value of the parameter μ_{N+1}, taken on the interval $0 < \mu_{N+1} < \mu^*_{N+1}$ and $\cos\alpha_{N,N+1} \neq 0$, the corresponding prediction function of unaccounted parameters will differ from zero, i.e., $\omega_{N+1}(\mu_{N+1}, \alpha_{N,N+1}) \neq 0$. Thus, choosing by desire the numerical values of unaccounted parameters function $\omega_\beta(\mu_{N+1}; \lambda_\beta, \alpha_{\beta-1,\beta}) =$
$\Omega_{N+1}(\lambda^*_{N+1}, \alpha_{N,N+1})$ corresponding to preceding small volumes $\Delta V_1, \Delta V_2, \ldots \Delta V_N$ and influencing by them beginning with the point $\mu_{N+1} = 0$ to the desired point μ^*_{N+1}, we get numerical values of predicting economic event
$Z_{N+1}(\mu^*_{N+1}; \lambda^*_{N+1}, \alpha_{N,N+1})$ on subsequent step of the small volume ΔV_{N+1} (Fig. 2).

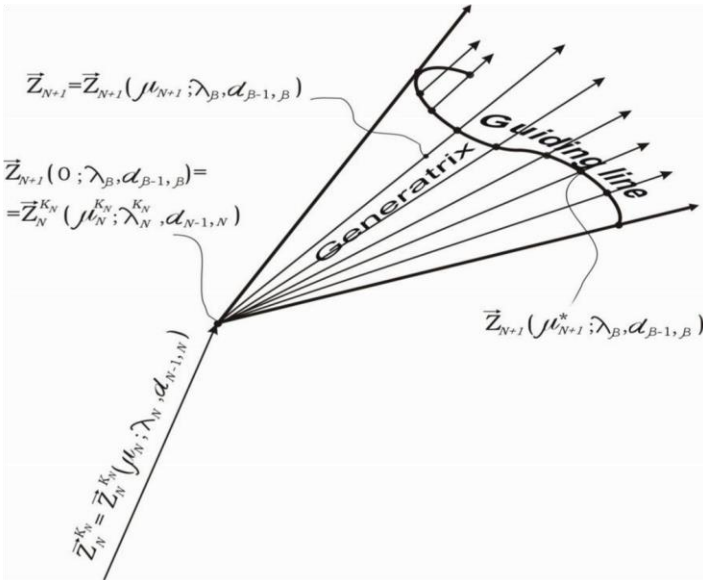


Fig. 2. The graph of prediction of process and its control at uncertainty conditions in finite-dimensional vector space. It is represented in the form of hypersonic surface whose points, of directrix will form the line of economic process prediction. Taking into account the fact that by desire we can choose the predicting influence function of unaccounted parameters □*N□1(□N* □1;□□N□1,□N,N□1) , then this function will represent a predicting control function of □unaccounted factors, and its appropriate function ZN* □1(□N* ,N□1;□□N□1,□N, N□1) will be a control aim function of economic event in finite-dimensional vector space. Speaking about unaccounted parameters prediction function
□N□1(□N□1;□N□1,□N ,N□1) we should understand their preliminarily calculated values in previous small volumes □V1,□V2,.....□VN of finitedimensional vector space. Therefore, in Eq. (24) we used calculated ready values of the function □N□1(□N□1;□N□1,□N ,N□1) .Thus, influencing by the unaccounted parameters influence functions of the form
□N□1(□N□1;□N□1,□N ,N□1) or by their combinations from the end of the vector equation of piecewise-linear straight □kN kN kN □ □
line zN (□N ; □N ,□N -1, N) situated on the boundary of the small volume ZN□1(□) □ Z N □1(□N □1;□N □1,□N , N □1) there will originate the vectors □VN and □VN□1, lying on the subsequent small volume □VN □1 .These vectors will represent the generators of hyperbolic surface of finite-dimensional vector space. The values of this series vectorfunctions for small values of the parameter □N□1 □□*N□1, i.e., Z□□N □1(□N* □1;□N □1,□N , N □1) will represent the points directrix of hyperconic surface of finite-dimensional vector space (Fig. 2). The series of the values of the points of directrix hyperconic surface will create a domain of change of predictable values of the function of □□ZN* □1(□N* □1;□□N□1,□N, N□1) in the further step in the small volume □VN □1 .This predictable function will have □□minimum and maximum of its values [ZN* □1(□N* □1;□□N□1,□N, N□1)]min and [ZN* □1(□N* □1;□□N□1,□N,N□1)]max . Thus, the □ found domain of change of predictable function of economic process in the form Z N □1(□N* □1;□□N □1,□N , N □1) , or in other words, the points of directrix of hyperbolic surface will represent the domain of economic process control in finite-dimensional vector-space.

III. 2-Component Piecewise-Linear Economic-Mathematical Model and Method of Multivariate Prediction of Economic Process With Regard to Unaccounted Factors Influence in 3-Dimensional Vector Space

In this article we give a number of practically important piecewise-linear economic-mathematical models with regard to unaccounted parameters influence factor in their-dimensional vector space. And by means of two- and three-component piecewise-linear models suggest an appropriate method of multivariant prediction of economic process in subsequent stages and its control then at uncertainty conditions in 3-dimensional vector space [6-11, 13-15].

Given a statistical table describing some economic process in the form of the points (vector) set $\{a_n\}$ of 3-dimensional vector space R_3. Here the numbers a_{ni} are the coordinates of the vector a_n ($a_{n1}, a_{n2}, a_{n3}, \ldots a_{ni}$). With the help of the points (vectors) a_n represent the set of statistical points in the vector form in the form of 2component piecewise-linear function [1–6]:

$$\vec{z}_1 = \vec{a}_1 + \lambda_1(\vec{a}_2 - \vec{a}_1) \quad (33)$$

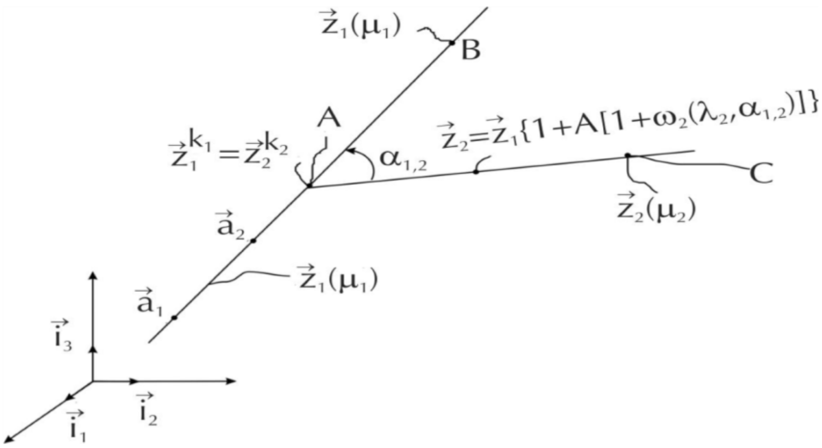
$$\vec{z}_2 = \vec{a}_2 + \lambda_2(\vec{a}_3 - \vec{a}_2) \quad (34)$$

Here $\vec{z}_1 = \vec{z}_1(z_{11}, z_{12}, z_{13})$ and $\vec{z}_2 = \vec{z}_2(z_{21}, z_{22}, z_{23})$ are the equations of the first and second piecewise-linear straight lines on 3-dimensional vector space; the vectors $\vec{a}_1(a_{11}, a_{12}, a_{13})$, $\vec{a}_2 = \vec{a}_2(a_{21}, a_{22}, a_{23})$ and $\vec{a}_3 = \vec{a}_3(a_{31}, a_{32}, a_{33})$ are the given points (vectors) in 3-dimensional space; $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are arbitrary parameters of the first and second piecewise-linear straight lines. And it holds the equality [illegible]; $\alpha_{1,2}$ is the angle between the piecewise-linear straight lines; k_1 is the intersection point between the first and second straight lines (Fig. 3). Note that in the general case, the intersection point of these straight lines may also not coincide with the point $\vec{a}_2$. Therefore, according to the conjugation condition $\vec{z}_1 = \vec{z}_2$, we denote the intersection point between the first and second piecewise-linear straight lines in 3-dimensional vector space by $\vec{z}_1^{\,k_1}$ (Fig. 3). Allowing for this fact, we write the equation of the second piecewise-linear straight line in the form Eq. (35):

$$\vec{z}_2 = \vec{z}_1^{\,k_1} + \lambda_2(\vec{a}_3 - \vec{z}_1^{\,k_1}) \quad (35)$$

where the value $\vec{z}_1^{\,k_1}$ is the value of the point (vector) of the first piecewise linear straight line at the k_1-the intersection point and equals:

Fig. 3. Construction of 2-component piecewise-linear economic-mathematical model



In 3-dimensional vector space R_3.

k1 k1 z1 a1 1 (a2 -a1) (36)

 k1 k1 k1

In particular case, z_1 a_2 for 1 1. In this case, the intersection point z_1 coincides with the point a_2. Now, according to Eqs. (1)–(11) of, the vector equation for the points of the second piecewise-linear straight line depending on the vector function of the first piecewise-linear straight line z_1 and introduced unaccounted parameters influence spatial function 2 (2 ,1,2) in 3-dimensional vector space is written in the form (Fig. 3) [7–9]:

$$z_2=z_1\{1+A\,[1+\omega_2(\lambda_2,\alpha_{1,2})]\}$$ (37)

Here

k1

a2 -a1 z1 -a 1

k1

A (1 -1) k1 (38)

z1(z1 -a1)

k1 k1 k1

2 z1 - z1 a3 - z1 z1(z1 -a1)

2 k1 k1 k1 (39)

1 -1z1(z1 - z1) a2 -a1z1 -a1

2 (2 ,1,2) 2cos1,2 (40)

 k1

k1 (a3 - z1)(a2 -a1) k1

μ2=(μ1 -μ1) k1 2 , for 1≥1 (41)

(a3 - z1)

Eq. (41) is the mathematical relation between arbitrary parameters 2 and 1. For the second piecewise-linear straight line, representing a straight line restricted with one end, condition Eq. (41) will hold for all 1≥1k1.

Furthermore, for the second intersection point k_2, i.e., for 2 2k2, the appropriate value of the parameter 1 will be determined as follows:

 k1 2

*k*2 *k*1 (*a*3 *z*1) *k* 2
1 1   *k*1   2 (42)
(*a*₃ *z*₁)(*a*₂ *a*₁)

The value of *cos*1,2 between the first and second piecewise-linear straight lines is determined by means of the scalar product of 2 vectors of the form (Fig. 3):

    *k*1   *k*1
AB *AC* (*z*1 *z*1)(*z*2 *z*1)
cos 1,2   *k*1   *k*1 (43)
AB *AC* *z*1 *z*1 *z*2 *z* 1

By calculating the values of *cos*1,2 we can use any values of arbitrary parameters 1 and 2.
Thus, in 3-dimensional vector space, determining the points (vectors):
∑3 ∑3 ∑3
*a*1 *a*1*mim* , *a*2 *a*2*mim* , *a*3 *a*3*mim*
*m*1 *m*1 *m*1
*k*1 ∑3 *z*1*km*1 *im* 3[*a*1*m* 1*k* 1(*a*2*m* -*a*1*m*)]*im* (44)
*z*1
*m*1 *m*1

Eq. (37) will represent an equation for the second vector straight line $z_2=z_2(\mu_1,\omega_2)$ depending on the unaccounted parameter influence function 2 (2 ,1,2) and arbitrary parameter $1 \geq \mu_1^{k_1}$. Represent the vector equation for the second piecewise-linear straight line Eq. (37) in the coordinate form. For that take into account that in 3dimensional space the vectors of the first and second piecewise-linear straight lines in the coordinate form are
represented in the form: *m*1
∑3 ∑3 *z*2 *z*2*m im* (45)
*z*₁ *z*₁*ₘ iₘ* and *m*1

In this case, the coordinates of the vector z_2 Eq. (37), i.e., z_{2m} will be expressed by the coordinates of the first piecewise-linear vector z_{1m}, spatial vector 2 and the unaccounted parameter influence function 2 (2 ,1,2) , in the form:

$z_{2m}=\{1+A[1\omega_2(\lambda_2,\alpha_{1,2})]\}z_{1m}$, for *m*1,2,3 (46)

Here the coordinate notation of the coefficients *A* , 2 and 2 (2 ,1,2) , by Eqs. (38)–(41), will be of the form:

$$\sum^{3}(a_{2i}-a_{1i})^2$$
A (1*k*1 -1) 3 *i*1 (47)

∑(*a*2*i* -*a*1*i*)[*a*1*i* 1(*a*2*i* -*a*1*i*)]
*i*1

$$\frac{2}{\mu_1^{k_1}-1}\left[\frac{\sqrt{\sum_{i=1}^{3}\{a_{3i}-[a_{1i}+\mu_1^{k}(a_{2i}-a_{1i})]\}^2}}{\sqrt{\sum_{i=1}^{3}(a_{2i}-a_{1i})^2}}\right.$$
2 (48)

□2 (□2 ,□1,2) □ □2cos□1,2 (49)

$$□_2 □(\mu_1 - \mu_1^{k_1}) \frac{\sum_{i=1}^{3}(a_{2i} - a_{1i})[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]}{\sum^{3}[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]^2}, \text{ for } □_1 □ □_1{}^{k1} \quad (50)$$

i□1

$$\sum^{3}(a_{2i} - a_{1i})[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]$$

□2k2 □ i□1 (□1k2 -□1k1) (51)

$$\sum^{3}[a_{3i} - a_{1i} - \mu_1^{k_1}(a_{2i} - a_{1i})]^2$$

i□1

Now, for the case economic process represented in the form of 2-component piecewise-linear economicmathematical model, investigate the prediction and control of such a process on the subsequent □V_3 (x1,x2 ,x3) small volume of 3-dimensional vector space with regard to unaccounted parameter influence function □2 (□2 ,□1,2) . And the value of the unaccounted parameter □2 (□2 ,□1,2) function is assumed to be known [6-11,13-15]. A method □□for constructing a predicting vector function of economic process Z_N□1(□) with regard to the introduced unaccounted parameters influence predicting function □N□1(□N□1,□N,N□1) in m-dimensional vector space, represented by Eqs. (24)–(30) was developed above. Apply this method to the case of the given 2-component piecewise-linear economic model 3-dimensional vector space. It will be of the form:

□ □□ □k2

Z_3 (1) □ z_1{1□ A[1□□2 (□2 ,□1,2)□□3 (□3,□2,3)]} (52)

Where

□2 (□k22 ,□1,2) □□k22 □cos□1,2 □

k2 z□1 □ z□1k1 □ a□3 □ z□1k1 z□1(z□1k1 □a□□1)

□2

□ K1 □ □□□□□k1 □ □□□□□k1 □cos□ 1,2 (53)

□1 □□1 z1(z1 □ z1) a2 □a1 □ z1 □a1

□ □□□ □□□□k1

(a □a)(a □ z)

□2 □ (□1 □□1^{k1})□ 2 □□1 □k31 2 1 , □1 □ □1^{k1} (54)

(a_3 □ z_1)

□ □□□k1 □□k1 a2 □a1 □ z1 □ a 1

□ A □ (□1 □□1)□ □□□k1 □ (55) □k □ □□k □□□ □

z_1(z_1 □ a_1)

□ z □ z□3 (□3 ,□2,3) □□3 □cos□2,3 (56) □ a (1)□ z

□3 □ K1 3 □ 2 □□2 2 □□4□k21 2

2 □ □□z1(□z1k1 □□$k$$a$1 1)

□ (57)

□1 □□1 z1(z2 □ z2) a2 □a1 □ z1 □a1

□ □k1 □ □□k2

(a □ z)(a (1)□ z)

□3 □ (□2 □□2k2)□ 3 □1 4□□k2 2 2 , □2 □ □2k2 , □3 □ 0 (58)

(a_4 (1)□ z_2)

Here, according to Eq. (40), the vector $\bar a_4(1)$ is of the form:

$$\bar a_4(1) = a_{41}(1) i_1 + a_{42}(1) i_2 + a_{43}(1) i_3 = \sum_{m=1}^{3} a_{4m}(1) i_m \qquad (59)$$

And the coordinates of a_{42} and a_{43} are expressed by the arbitrarily given coordinate $a_{41} \neq z_{21}^{k2}$ in the form:

$$\frac{a_{41}(1) - z_{21}^{k2}}{a_{21} - a_{11}} = \frac{a_{42}(1) - z_{22}^{k2}}{a_{22} - a_{12}} = \frac{a_{43}(1) - z_{23}^{k2}}{a_{23} - a_{13}} = C \qquad (60)$$

Hence:

$$a_{42}(1) = z_{22}^{k2} + \frac{a_{22} - a_{12}}{a_{21} - a_{11}} (a_{41}(1) - z_{21}^{k2})$$

$$a_{43}(1) = z_{23}^{k3} + \frac{a_{23} - a_{13}}{a_{21} - a_{11}} (a_{41}(1) - z_{21}^{k2}) \qquad (61)$$

Here the coefficients a_{2m}, a_{1m} and z_{2m} are the coordinates of the vectors $\bar a_1, \bar a_2, \bar z_2$ in 3-dimensional vector space and equal:

$$\bar a_1 = \sum_{m=1}^{3} a_{1m} i_m, \quad \bar a_2 = \sum_{m=1}^{3} a_{2m} i_m, \quad \bar z_2^{k2} = \sum_{m=1}^{3} z_{2m}^{k2} i_m \qquad (62)$$

Note that in the vectors $Z_3(1)$ and $\bar a_4(1)$ the index (1) in the brackets means that the vector $Z_3(1)$ is parallel to the first piecewise-linear vector function $\bar z_1$. This means that the economic process beginning with the point $\bar z_2^{k2}$ will hold by the scenario of the first piecewise-linear equation (Fig. 4).

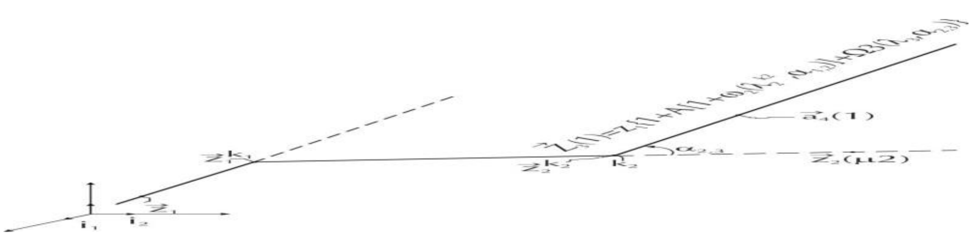


Fig. 4. Construction of the predicting vector function $Z_3(\mu)$ with regard to unaccounted parameter influence predicting function $\Omega_3(\mu_3, \alpha_{2,3})$ on the base of 2-component economic-mathematical model in 3-dimensional vector space R_3.

The expression of $\cos\alpha_{2,3}$ corresponding to the cosine of the angle between the second piecewise-linear straight line $\bar z_2$ and the predicting third vector straight line $Z_3(1)$ on the base of the scalar product of 2 vectors, is represented in the form (Fig. 4):

$$(\bar z_2 - \bar z_2^{k2}) \cdot (\bar a_4(1) - \bar z_2^{k2})$$

cos ?$_{2,3}$? ? ?$k2$ | ? ‖ ?$k2$ | (63)
$z2$? $z2$ $a4$ (1)? z 2 ‖ |

IV. Results Method of Numerical Calculation of 2-Component Economic-Mathematical Model and Definition of Predicting Vector Function with Regard to Unaccounted Factors Influence in 3Dimensional Vector Space

Below we have given the numerical construction of a 2-component piecewise-linear economic mathematical model, and by means of the given model will determine the predicting function on the subsequent third small volume of the investigated economic process in 3-dimensional vector space [6-11,13-15]. Given a statistical table describing ?
some economic process in the form of the points (vectors) set $\{a_n\}$ in 3-dimensional vector space R_3 . Represent the ?
set of vectors $\{a_n\}$ of statistical values in the form of adjacent 2-component piecewise-linear vector equation of the form Eq. (32):

? ?$k1$??????$k1$ z_2 ? z_1 ??$_2$ (a_3 - z_1) (64)

? ? ? ?
where z_1 ? z_1(z_{11},z_{12},z_{13}) and z_2 ? z_2(z_{21},z_{22},z_{23}) are the equations of the first and second piecewise-
? ? ?
linear straight lines in 3-dimensional vector space; the vectors a_1(a_{11},a_{12},a_{13}) , a_2 ?a_2(a_{21},a_{22},a_{23}) and
? ?
$a3$??$a3$(?$a31$?,a?32,$a33$?) are given points (vectors) in 3? ? -dimensional space of the form:
?
$a1$? $i1$?? $i2$??$i3$ a??2 ? 3$i1$? 2$i2$? 4,5$i3$ (65)
?
a_3 ? 6i_1 ? 4i_2 ? 7i_3 (66)

?$_1$ ≥ 0 and ?$_2$ ≥ 0 are arbitrary parameter. Substituting Eq. (65) and (66) in Eq. (3264), the coordinate form of the vector equation of the first vector straight line will accept the form:?????
?
z_1 ? (1? 2?$_1$)i_1 ? (1??$_1$)i_2 ? (1?3,5?$_1$)i_3 (67)

?^{k1} ?^{k1} ????^{k1}
As the intersection point of 2 straight lines z_1 that should satisfy the conjugation condition z_1 ? z_2 may
?? ?$k1$ also not coincide with the point a_2 , then its appropriate value of the parameter ?$_1$ will be ?$_1$?1. In this connection, in numerical calculation, we accept the value of the parameter ?$_1^{k1}$ for the intersection point between
$k1$??^{k1} piecewise-linear straight lines equal 1.5, i.e., ?$_1$?1,5 . Then the value of the intersection point z_1 Eq. (67) will equal: ? ? ?
?$k1$
z_1 ? 4i_1 ? 2,5i_2 ? 6,25i_3 (68)

By Eq. (37) the equation of the second straight line in the vector form is expressed by the vector equation of the first ?
piecewise-linear straight line z_1 of the form Eq. (67) and the unaccounted parameter function ?$_2$ (?$_2$,?$_{1,2}$) in the form:
? ?

z2 □ z1{1□ A[1□□2 (□2 ,□1,2)]} (69)

Here the coefficient A and the unaccounted parameter function □2 (□2 ,□1,2) of the economic process will be of the form Eqs. (38)–(41) and (1143):

□□□□□k1 □

1 a2 -a1 z1 -a 1 k1

A □ (□1k -□1) □□□k1 □ for □1≥□1 □1,5 (70) z1(z1 -a1)

ω2 (λk22 , α1,2) □ λk22 cosα1 , 2 (71)

2 □□□k1 □ z□□1k1 z□1(z□1k1 - a□1)

□2k z1 - z1 a3 -

□2 □ k1 □□□□□k1 □□□□k1 □(72)

□1 -□1z1(z1 - z1) a2 -a1z1 - a1

□ □k1 □ □□k1

(z1 □ z1)(z2 □ z1)

cos □1,2 □ □ □k1 □ □k1 (73)

z1 □ z1 z2 □ z 1

Here the parameter □2 corresponding to the points of the second piecewise-linear straight line is connected

k1 □□with the appropriate parameter □1 by Eq. (41). Here for the values □1 □ □1 □1,5. In Eq. (73) the vector z2 is □□calculated by Eq. (65) for any value of □2 in the interval 0 □□2 □1, and the vector z1 is of the form Eq. (64) for

k1 □□any value of □2 □ □1 . By calculating the value of the expression Cos□1,2 by Eq. (73), the value of z1 may be □calculated for the value of a3 or for □2 that corresponds to the value of the second intersection point k2, i.e., for

□□ □2k2 . Substituting the value of the parameter □1k1 □ 1,5 , and also Eq. (3466) in Eq. (41), set up a numerical relation between the parameters □2 and □1 in the form:

□2 □1,1927(□1 -1,5) for □1 □1,5; 0 □ □2 □ □2k2 □1 (74)

Thus, (74) is the numerical representation of mathematical relation between the parameters □1 and □2 . Defining any value of □2 □ 0 by Eq. (74), it is easy to determine the appropriate value of the parameter □1. From (74) it will follow:

□1 □1,5□0,8384□2 (74a)

Calculate the values of the coefficient A , the unaccounted parameter function □2 (□2 ,□1,2) of economic process and cos□1,2 . For that, substituting Eqs. (66)–(67) in Eq. (74), and also the numerical value of the parameter

□1k1 □1,5 in Eqs. (70)–(73), define the numerical values of A , □2(□2,□1,2) and Cos□1,2 for □1 □ 1,5 in the form:

25,8751

A □ □(□1 □1,5) (75)

9,75□ 25,875□□ 1

9,75□25,875□ 1 2

□2 □0,1208 2 38,8125□51,25□1 □17,25□1 (76)

9,75□19,375□10□17,25□ 1

cos□1,2 =0.8495 (77)

Numerical values of A and □2 for the second intersection point, i.e., for □1 □ 3,1768 calculated by Eqs. (75) and

(76) will be equal to:

A(3,1768) □□0,4719 , □2 □ (3,1768) □□0,7495

Substituting Eqs. (67), (75)–(77) in Eq. (69), find the equation of the second vector straight line in the vector form depending on the vector function of the first piecewise-linear straight line and appropriate for the second linear straight line of the parameter $\mu_1 \geq 1{,}5$ in the form (Fig. 5):

$$\vec{z}_2 = \varphi_0(\mu_1)\cdot\vec{z}_1 + \varphi_0(\mu_1)\cdot[(1+2\mu_1)\vec{i}_1 + (1+\mu_1)\vec{i}_2 + (1+3{,}5\mu_1)\vec{i}_3] \text{ for } \mu_1 \geq 1{,}5 \quad (78)$$

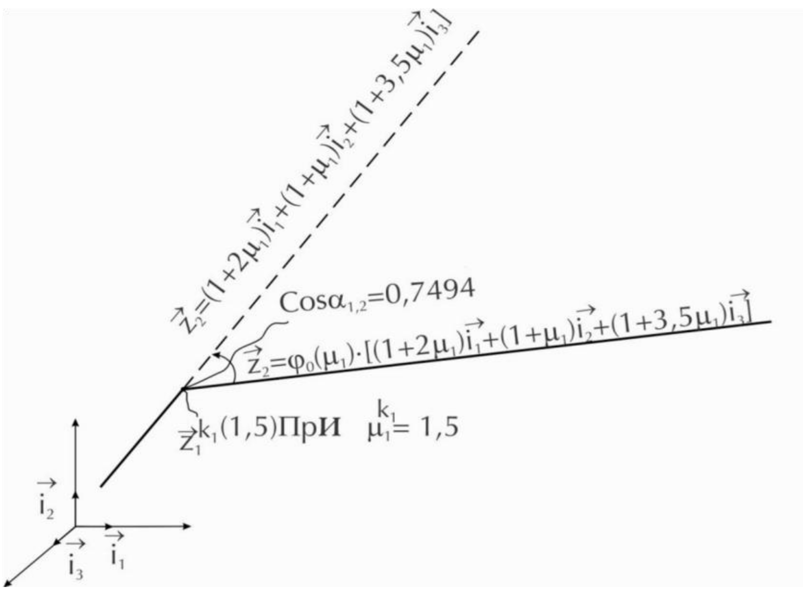
Here

$$\varphi_0(\mu_1) = 1 - (\mu_1 - 1{,}5)\left[9{,}75 + 25{,}875\,\mu_1 - 1 - 0{,}1026\,\frac{9{,}75 + 25{,}875\,\mu_1}{9{,}75 + 19{,}375\,\mu_1 + 17{,}25\,\mu_1^2}\sqrt{38{,}8125 + 51{,}75\,\mu_1 + 17{,}25\,\mu_1^2}\right] \quad (79)$$

Numerical values $\varphi_0(\mu_1)$ at the second intersection point, i.e., for $\mu_1 = 3{,}1768$ will equal:

$\varphi_0(3{,}1768) = 0{,}8297$

Fig. 5. Numerical representation of 2-component piecewise-linear economic-mathematical model in 3dimensional vector space R_3.



Now investigate the problem of prediction and control of economic process in the subsequent $\Delta V_3(x_1, x_2, x_3)$ volume of 3-dimensiona vector space with regard to unaccounted parameters factor that hold on preceding states of the process [6-11,13-15].Above for the case of 2-component piecewise-linear straight line it was numerically constructed the second vector straight line (78) depending on an arbitrary parameter μ_1 and unaccounted parameter influence space function $\varphi_2(\mu_2, \mu_{1,2})$. On the other hand, for the 2-component case economic process a predicting vector function $\vec{Z}_3(1)$ with regard to the introduced unaccounted parameter influence predicting function $\varphi_3(\mu_3, \mu_{2,3})$ was suggested in the form:

$Z_3(1)$ � $z_1\{1$� $A[1$��$_2$ (�$_2$,�$_{1,2}$)��$_3$ (�$_3$,�$_{2,3}$)]} (80)

Here the coefficient A , the unaccounted parameter function $\omega_2(\lambda_2^{k2}, \alpha_{1,2})$, and also the unaccounted parameter predicting function �$_3$ (�$_3$,�$_{2,3}$) are of the form Eqs. (53)–(58) define numerical values of these expressions. As the �

economic process predicting function $Z_3(1)$ is the third piecewise-linear function, at first we define the value of the

����^{k2} vector function z_2 at the second intersection point k_2. The parameter �$_2$ acting on the segment of the second piecewise-linear straight line changes in the interval 0 � �$_2$ � �$_2^{k2}$ �1. Here the value of the parameter �$_2^{k2}$ belongs to the intersection point between the second and third straight lines. According to approximation of statistical points, this point should be defined. Therefore, giving the value of the parameter �$_2^{k2}$ at the second intersection point k_2, define from Eq. (41) the appropriate value of the parameter �$_1^{k2}$, in the form:

� ����$k1$ 2

$k2$ $k1$ $k2$ $(a3$ � $z1)$

�1 � �1 ��2 � �$k1$ � � (81)

$(a_3$ � $z_1)(a_2$ �$a_1)$

For conducting numerical calculation we accept �$_2^{k2}$ � 2. For the value of the parameter �$_2^{k2}$ � 2, we define the appropriate numerical value of the parameter �$_1$, that will be denoted by �$_1^{k2}$, from Eq. (81) or Eq. (74). It will equal:

�$_1^{k2}$ � 3,1768 (82)

Thus, we established the range of the parameter �$_1$ corresponding to the change of the parameter �$_2$ of the segment of the second piecewise-linear straight line, in the form:

1,5 ��$_1$ � 3,1768 for 0 � �$_2$ � �$_2^{k2}$ � 2 (83)

Though Eq. (81) is valid for the values of the parameter �$_2$ � 2 as well. In this case, the value of the prediction �

function $Z_3^{k2}(1)$ at the intersection point k_2 , i.e., for �$_3$ � 0, �$_2$ � 2 , �$_1^{k2}$ � 3,1768 coincides with the value of the function of the second piecewise-linear straight line:

��$k2$ ����$k2$

$Z_3(1) = z_2$ (84)

Note that at the intersection point k_2, i.e., for �$_2^{k2}$ � 2, �$_3^{k2}$ � 0 the unaccounted parameters influence predicting �

function �$_3$ (�$_3$,�$_{23}$) � 0.But the function z_2 has the form (78). Therefore, it suffices to substitute to Eq. (78) the �

value of the parameter �$_1^{k2}$ � 3,1768 that will be defined both as the value of the predicting function Z_3^{k2} (1) at the

��^{k2} initial point �$_2$ � 2 �$_3$ � 0 of the third vector straight line and the value of the point z_2 at the final point of the second piecewise-linear straight line at the point κ$_2$, in the form: k_2 � � �

Z_3 (1)�1�$_{3,1768}$ � 6,1013i_1 �3,4655i_2 �10,055i_3

for �$_2$ � 2 , �$_1^{k2}$ � 3,1768, �$_3$ � 0 (85)

� �

Calculate the point $a_4(1)$. For that give in an arbitrary form 1 of the coordinates of the vector $a_4(1)$, for instance, ��the coordinate $a_{41}(1)$, and by Eq. (61) calculate the remaining coordinates of the vector $a_4(1)$. Furthermore, $a_{41}(1)$ is given so that $a_{41}(1)$ were greater than the coordinates z_{21}^{k2} � 5,8411. Therefore accept the value $a_{41}(1)$ =6,5. In ��this case, substituting Eqs. (66) and (85) in Eq. (61), define the vector $a_4(1)$ in the coordinate form depending on an arbitrarily given value of $a_{41}(1)$ in the form:

� � 1 � �

$a_4(1)$ ⍰ $a_{41}i_1$ ⍰ (1,3707⍰ $a_{41})i_2$ ⍰ (⍰3,5163⍰ 2,25$a_{41})i_3$ (86)

3
⍰
For the value $a_{41}(1)$ =six,5, the vector accepts the form $a_4(1)$:
⍰ ⍰ ⍰ ⍰
$a_4(1)$ ⍰ 6,5i_1 ⍰ 3,5374i_2 ⍰11,1087i_3 (87)

For numerical definition of the coefficient A the unaccounted parameter function $\omega_2(\lambda_2^{k2}, \alpha_{1,2})$ and also the unaccounted parameter predicting function ⍰$_3$ (⍰$_3$,⍰$_{2,3}$) allowing for Eqs. (66)–(68), (74), (79), (41) and (85) conduct the following calculations:

⍰^{k1} ⍰ ⍰ ⍰ ⍰⍰⍰⍰⍰⍰⍰
1) z_1 ⍰a_1 ⍰ 4i_1 ⍰ 2,5i_2 ⍰6,25i_3 ⍰i_1 ⍰i_2 ⍰i_3 ⍰ 6,23 (88)
⍰ ⍰ ⍰⍰⍰ ⍰
2) a_2 ⍰a_1 ⍰ 2i_1 ⍰i_2 ⍰3,5i_3⍰ 4,1533 (89)
⍰⍰⍰$_k$1 ⍰ ⍰ ⍰⍰ ⍰⍰ ⍰
3) z_1(⍰z_1 ⍰ a_1)⍰⍰⍰ (1⍰ 2⍰$_1$)i_1 ⍰ (1⍰ ⍰$_1$)i_2 ⍰ (1⍰ 3,5⍰$_1$)i_3 (3i_1 ⍰
⍰1,5i_2 ⍰ 5,25i_3) ⍰ 9.75⍰25,875⍰$_1$= A_1(⍰$_1$) (90)
⍰⍰ ⍰$_k$2 ⍰ ⍰ ⍰ ⍰ ⍰⍰⍰⍰⍰
4) z_2 ⍰ z_2⍰⍰⍰ {⍰$_0$(μ$_1$⍰) (1⍰ 2μ$_1$)i_1⍰⍰⍰ (1⍰μ$_1$)i_2 ⍰ (1⍰3,5μ$_1$)i_3 ⍰
⍰5,8411i_1 ⍰3,3177i_2 ⍰9,6262i_3}=
⍰ ⍰
=⍰⍰$_0$ (1⍰ 2⍰$_1$) ⍰5,8411⍰i_1 ⍰ ⍰⍰$_0$ (1⍰⍰$_1$) ⍰3,3177⍰i_2 +
⍰
+⍰⍰$_0$ (1⍰3,5⍰$_1$) ⍰9,^{6262}⍰i_3 (91)
⍰ ⍰$_k$2 ⍰ ⍰ ⍰ ⍰⍰⍰⍰⍰
5) z_2 ⍰ z_2 ⍰ ⍰$_0$ (⍰$_1$) (1⍰ 2⍰$_1$)i_1 ⍰ (1⍰⍰$_1$)i_2 ⍰(1⍰3,5⍰$_1$)i_3 –
⍰ ⍰ ⍰
–5,8411i_1 ⍰3,3177i_2 ⍰9,6262i_3 =
⍰⍰$_0$ (1⍰ 2⍰$_1$)⍰5,8411⍰^{2} ⍰⍰⍰$_0$ (1⍰⍰$_1$)⍰3,3177⍰^{2} ⍰
= √ = A_2(⍰$_1$) (92)
2
⍰⍰⍰$_0$ (1⍰3,5⍰$_1$)⍰9,^{6262}⍰
⍰⍰⍰ ⍰⍰k2 2 2 2
6) $z1(z2$ ⍰ $z2$) =⍰0 (⍰1)[(1⍰ 2⍰1) ⍰(1⍰⍰1) ⍰(1⍰3,5⍰1)]⍰

7) $\left|\vec{a}_4(1) - \vec{z}_2^{k_2}\right| =$ −[18,785−48,6916α_1]= $A_4(\alpha_1)$ (93)

$= \left|a_{41}\vec{i}_1 + (1{,}3707 - \frac{1}{3}a_{41}\vec{i}_2 + (-3{,}5163 + 2{,}25a_{41})\vec{i}_3 -\right.$

$\left.-5{,}8411\vec{i}_1 - 3{,}3177\vec{i}_2 - 9{,}6262\vec{i}_3\right| =$

$\sqrt{(a_{41}(1) - 5{,}8411)^2 + (-1{,}947 - \frac{1}{3}a_{41}(1))^2 + (-13{,}1425 + 2{,}25a_{41}(1))^2}$

8) $[\vec{a}_4(1) - \vec{z}_2^{k_2}]^2 =$

–

= = $A_3(\alpha_1)$ (94)

$=(a_{41}(1)-5{,}8411)^2 + (-1{,}947 - \frac{1}{3}a_{41}(1))^2 +$

$+(-13{,}1425+2{,}25a_{41}(1))^2$ (95)

9) $(\vec{a}_3 - \vec{z}_1^{k1})(\vec{a}_4(1) - \vec{z}_2^{k2}) =$

$= 2(a_{41}(1)-6{,}1013)-1{,}5(-1{,}947 - \frac{1}{3}a_{41}a_{41}(1))+$

$+0{,}75(-13{,}1425+ 2{,}25a_{41}(1))$ (96)

Substituting the values $a_{41}(1)$ =6.5 and Eq. (86) in Eqs. (86)–(98), we have:

$\left|\vec{a}_4(1) - \vec{z}_2^{k2}\right| =1{,}9929$ (97)

$[\vec{a}_4(1) - \vec{z}_2^{k2}]^2 =3{,}9715$ (98)

$(\vec{a}_3 - \vec{z}_1^{k1})(\vec{a}_4(1) - \vec{z}_2^{k2}) =2{,}5532$ (99)

Now set up numerical relation between the parameters α_3 and α_1. For that, substituting Eqs. (97)–(99), and taking into account the numerical values $a_{41}(1) = 6{,}5$ and $\alpha_2^{k2} = 2$, the relation Eq. (58) between the parameters will be of the form:

$\alpha_3 = \frac{2{,}5532}{3{,}9715}(\alpha_2 - 2)$ for $\alpha_2 \geq 2$, $\alpha_3 \geq 0$ or $\alpha_3 = 1{,}6429(\alpha_2 - 2)$ (100)

Substituting the numerical dependence between the parameters α_2 and α_1 in the form Eq. (74) in (100), set up dependence of the parameter α_3 on the parameter α_1 in the form:

$\alpha_3 = 0{,}7668(\alpha_1 - 3{,}1768)$ for $\alpha_1 \geq 3{,}1768$ (100a) or

$\alpha_1 = 1{,}3041\alpha_3 + 3{,}1768$ for $\alpha_3 \geq 0$

Now, substituting Eqs. (88), (89), (90), (92), (93), (94), and (100) in Eq. (57), define the unaccounted factors predicting parameter α_3 in the form:

$\alpha^1 - 3{,}1768\ A^1(\alpha^1)A^2(\alpha^1)A^3(\alpha^1)$

$\alpha_3 = -0{,}0296 -$ for $\alpha_1 \geq 3{,}1768$ (101) $\alpha_1 - 1{,}5\ A_4(\alpha_1)$

where 0 (1) is of the form Eq. (79).

Now, by Eq. (63), calculate the cosine of the angle *cos*23 between the economic process predicting vector function

Z3 (1) and the second piecewise-linear vector-function z2(2) in the form (Fig. 6.):

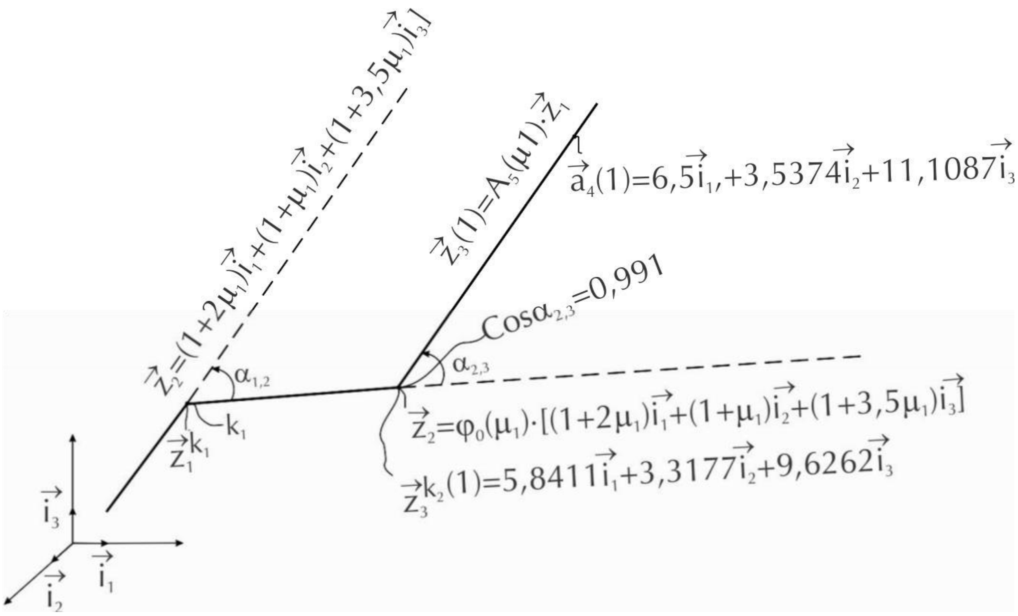
k2   k2

(a4 (1) z2)(z2 (2) z2)

cos 2,3   k2   k2 (102)

a4 (1) z2   z2 (2) z 2

Fig. 6. Numerical construction of predicting vector function Z3 () on the base of 2-component economic-mathematical model in 3-dimensional vector space R3 .



Taking into account Eqs. (91)–(95), expression of cos2,3 takes the form:

cos2,3 =

(6,72631 2,3611)0 (1)18,8484

=

0 (1 21)5,84112 0 (11)3,31772

1,6372

0 (13,51)9,62622

For 1 5 the numerical value of cos2,3 will be:

cos2,3 0,9448 (103)

From Eq. (2456) calculate 3 (3 ,2,3). For that substitute Eqs. (90)–(100), (92), (6294), (100), (103) in Eq. (56), and calculate 3 (3 ,2,3):

1 3,1768 A1(1)A2 (1)A3 (1)

3 (3 ,2,3) 0,028 ,

1,51A 4

For

□1 □ 3,1768 (104)

Now calculate the unaccounted parameter function □2 (□^{k}2^{2} ,□1,2) belonging to the second piecewise-linear straight line, and take into account the character of relation between the parameters □2 and □1 given in the form Eq. (74):

□2 □1,1927(□1 □1,5) for □1 □1,5, 0 □□2 □□2^{k2} □1 (105)

Hence:

□1 □1,5□0,8384□2 (106)

For □2 □ □2^{k2} from Eq. (106):

□1^{k2} □1,5□0,8384□2 ^{k2} (107)

For the considered example, for the second intersection point k_2 the value of the parameter □2^{k2} earlier was accepted to be equal to 2, i.e., □2^{k2} □ 2. In this case, the appropriate numerical value of the parameter □1^{k2} by Eq. (107) will equal:

□1 ^{k2} =3,1768 (108)

$k2$ □

Now carry out appropriate calculations by Eq. (53) for defining □2 (□2 ,□1,2) , and calculate the vector z_1(□1) in it

for the value of the parameter □1 □ □1^{k2} □ 3,1768 . Taking into account □1^{k1} □1,5 , □2^{k2} □ 2, □1^{k2} =3,1768, *cos*□1,2 □ 0,8495, and also Eqs. (45), (56)–(58), define the numerical value of □2 (□^{k}2^{2} ,□1,2) in the form:

□2 (□^{k}2^{2} ,□1,2) □□^{k}2^{2} □*cos*□12 □ □0,635 (109)

Substituting Eqs. (88)–(90) in Eq. (55), express the coefficient A by the parameter □1 □ □1^{k2} □ 3,1768 in the form:

□^{1} □1,5 $k2$

A □ □25,875 for □1 □ □1 □ 3,1768 (109a)

A_1(□1)

where

A_1(□1) □ ^{1},75□25,875□ 1

Substituting the numerical values of the coefficient A Eq. (109a), the unaccounted parameter influence function □2 (□^{k}2^{2} ,□1,2) Eq. (109) and also the unaccounted parameter influence predicting function □3 (□3 ,□2,3) Eq. (104) in Eq. (52), for the case of 2-component piecewise-linear straight line find the form of the economic □

process predicting vector function Z_3(1) in 3-dimensional vector space in the form (Fig. 6) [4–6]:

□ □□□ □1 □1,5

Z_3 (1) □ z_1 □1□9,4444 □[1□

□ $A1$(□1)

^{1} ,75□ 25,875□Π_1 □1□

□

$$+0,1026\frac{9,75+25,875\cdot\mu_1}{9,75+19,375\cdot\mu_1-17,25\mu_1^2}\sqrt{\begin{matrix}38,8125-51,75\cdot\mu_1+\\+17,25\mu_1^2\end{matrix}}\Bigg] \qquad (116)$$

Eq. (78) is written in the compact form as follows (Fig. 7):

□ □3,1768 A (□)A (□)A (□)□

□0,0767 1 □ 1 1 2 □1 □ 3,1768, (110)

1 3 1 □ for □1 □1,5 $A4$

(□1) □

where □ □ □

□

z_1 □ (1□ 2□1)i_1 □ (1□□1)i_2 □ (1□3,5□1)i_3 (111)

$A_1(□_1) = 9.75 □ 25,875□_1$ (112)

$A_2(□_1) = \sqrt{[□_0(1□2□_1)□5,8411[^2 □ [□_0(1□□_1)□3,3177[^2 □ □ [□_0(1□3,5□_1)□9,6262[^2}$ (113)

(114)

□[18,785□ 48,6916□1]

(115)

$A_3(□_1) = \sqrt{(a_{41} □ 5,8411)^2 □ (□1,947 □ \frac{1}{3} a_{41})^2 □ □ (□13,1425 □ 2,25 a_{41})^2}$

□

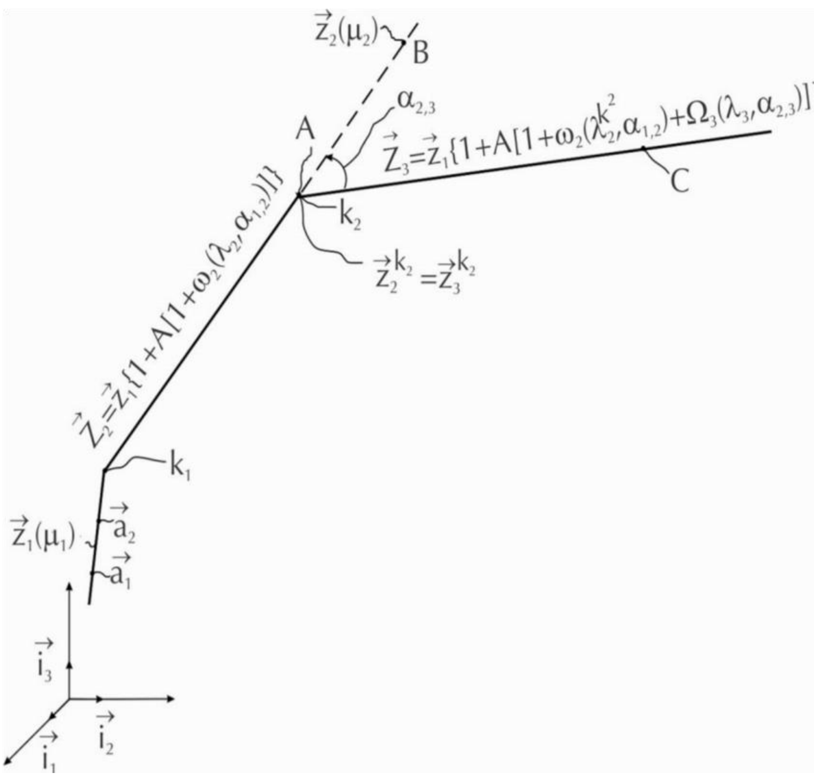
□

25,8751 □

$A_4(□_1) = □_0(□_1)[(1□2□_1)^2 □ (1□□_1)^2 □ (1□3,5□_1)^2] □$

□0 (□1) □1□(□1 -1,5)□

□□**Fig. 7. Compact form of representation of numerical expression of the predicting vector function Z_3 (□) constructed on the base of 2-component model in 3-dimensional vector space R_3.**



□ □

Z_3 (1) □ A_5 (□1)□ z_1 (117) where

□[1] □1,5

A_5 (□1) □1□9,4444 □[1□

$A1$(?1)
?¹ ?3,1768 A^1(?¹)A^2 (?¹)A^3 (?¹)
?0,0767 ? for ?₁ ? 3,1768 (118)

?₁ ?1,5 A_4 (?₁)

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