

## PARTICLE FILTER-BASED APPROACH FOR ENHANCING ACCURACY OF THREE-AXIS FLUXGATE MAGNETOMETERS

**Liang, Xinyu**

Shanghai Research Institute of Chemical Industry Co., Ltd, 200062, Shanghai, China

### **Abstract:**

*With the wide application of fluxgate magnetometer sensor in the field of high-precision magnetic field measurement, the inconsistency between its three axes due to production process problems has attracted more and more attention. The purpose of this paper is to correct the measurement errors of the three axes of the sensor respectively, so as to improve the measurement accuracy of the magnetometer. In this paper, the particle filtering algorithm is used as the core, based on the error model of the fluxgate magnetometer, in which the nine correction parameters are updated and iterated, and after the importance resampling process, the parameter estimates are obtained and substituted into the error correction model, and then the error correction of the three-axis fluxgate is realized. The proposed method is verified by simulation and experimental data. The results show that the proposed method can effectively correct the triaxial error and improve the working performance of the sensor. Particle filter algorithm is an algorithm based on the spatial model of dynamic system, which has good filtering effect on nonlinear system. The research results of this paper are verified by examples, which is of great significance to improve and improve the measurement accuracy of fluxgate.*

**Keywords:** Fluxgate magnetometer, Error correction, Particle filter

### **1. Introduction**

Fluxgate magnetometer sensors have gradually been widely used in the field of high-precision magnetic field measurement due to their advantages such as high resolution, simple structure, small size and good temperature stability [1-3]. Fluxgate magnetometer has gradually become the key equipment in magnetic inspection, magnetic field monitoring, earth magnetic field measurement, ship degaussing and other fields. However, in the actual use process, due to the limitation of the manufacturing technology, there are various types of errors between the three orthogonal axes of the fluxgate magnetometer, which makes a large deviation between the measured value of the sensor and the actual value, and seriously affects the measurement accuracy and reliability [4]. Therefore, it is necessary to correct the magnetometer before use to reduce the measurement error.

At present, considerable research has been carried out at home and abroad to solve this problem, which is mainly divided into two ideas in terms of methods: vector correction and scalar correction. In document [5], a vector compensation method is proposed to suppress the sensor measurement error by building a system composed of a cross magnetometer array, magnets, steel blocks and a zero-magnetic turntable. Vector correction needs to be based on a zero-magnetic turntable to monitor the attitude of the magnetometer in real time. This kind of correction method has high accuracy, but the experimental steps are cumbersome, which is more suitable for sensor correction in the laboratory environment. In contrast, scalar correction measures the magnetic field change data under different postures by placing the sensor in a uniform magnetic field, and estimates the correction parameters to achieve the purpose of error correction. Compared with vector correction, this kind of method has less computation and greatly simplified experimental steps, so it has become a more common method at present. In reference [6], by analyzing the non-orthogonality error of the three-component fluxgate sensor, the mathematical model description is given,

and a correction method based on the real coded genetic algorithm is proposed. The disadvantage of this kind of algorithm is that its correction effect is greatly affected by the initial parameters. In reference [7], a magnetic field component gradiometer error correction method based on function linked neural network and least square method is proposed by establishing a measurement error model. The calculation process of this method is complex and the accuracy is not high. Aiming at the problems of low accuracy and large influence of initial value in the current magnetometer correction methods, in this paper, we propose an axis flux gate error correction method based on particle filter algorithm. Particle filter algorithm is an algorithm based on the spatial model of dynamic system and has a good filtering effect on non-linear systems. It has unique advantages in processing the state and parameter estimation of non-Gaussian non-linear systems [8].

The rest of this paper is organized as follows: Section 2 introduces the fluxgate magnetometer error model. Section 3 introduces the research methods. Section 4 verifies the effectiveness of the proposed method through simulation and experiments, and discusses the results. The fifth part gives the conclusion.

## 2. Fluxgate error model

Fluxgate error model for fluxgate magnetometers, the measurement error mainly consists of zero bias error, scale coefficient error and three-axis non orthogonal error. Zero bias error is also called translation error, which is equivalent to superimposing an additional magnetic field on three orthogonal axes. Assuming that the zero bias errors of the three axes of the magnetometer are  $e_x$ ,  $e_y$  and  $e_z$ , the zero bias error can be expressed by formula (1):

$$B_{xo} = B_x + e_x$$

?

$$B_{yo} = B_y + e_y \quad (1)$$

?

$$B_{zo} = B_z + e_z$$

Where  $B_{xo}$ ,  $B_{yo}$  and  $B_{zo}$  are the measured values of the magnetometer, and  $B_x$ ,  $B_y$  and  $B_z$  are the actual values of the background uniform magnetic field.

The scale coefficient error is an error caused by the inconsistent sensitivities of the three measuring axes of the sensor. Assuming that the ratio of the output of the magnetometer to the standard value are  $k_1$ ,  $k_2$  and  $k_3$  respectively, the scale coefficient error of the magnetometer can be expressed by formula (2):

$$B_{xo} = k_1 B_x$$

?

$$B_{yo} = k_2 B_y \quad (2)$$

?

$$B_{zo} = k_3 B_z$$

In order to make the output of the three-axis magnetometer meet the requirements of the Cartesian coordinate system, the three axes must be installed orthogonally to each other. However, the actual threeaxis magnetometer is not completely orthogonal due to the limitation of processing and installation accuracy. Then the output of the three-axis magnetometer is not the three magnetic field components in the orthogonal coordinate system, and must be compensated. Let the three axes of the three-axis magnetometer be  $X$ ,  $Y$ ,  $Z$ . The outputs are  $B_x$ ,  $B_y$ ,  $B_z$ . Take the orthogonal coordinate system

$x_o$ ,  $y_o$ ,  $z_o$  and  $X$ ,  $Y$ ,  $Z$  coincide with the coordinate origin,  $Z$  axis coincides with  $z_o$ ,  $Y$  axis is coplanar with  $O$ ,  $y_o$  and  $z_o$ .  $\alpha$ ,  $\beta$ ,  $\gamma$  respectively represent the included angle between  $Y$  axis and  $y_o$  axis,  $X$  axis and  $O$ ,  $y_o$ ,  $x_o$  plane, the projection of  $X$  axis on the  $O$ ,  $y_o$ ,  $x_o$  plane and  $x_o$ . Based on coordinate transformation:

$$B_{xo} = B_x \cos(\beta) \cos(\gamma)$$

?

$$B_{yo} = B_y \cos(\alpha) + B_x \cos(\beta) \sin(\gamma) \quad (3)$$

?

$$B_{zo} = B_z + B_y \sin(\alpha) + B_x \sin(\beta)$$

Since the non-orthogonal angle of the magnetometer is very small,  $\alpha$ ,  $\beta$ , and  $\gamma$  are small and close to zero, it can be assumed that:

$$\sin(\alpha) = \alpha$$

$$\sin(\beta) = \beta \quad (4)$$

$$\sin(\gamma) = \gamma \cos(\alpha) = \cos(\beta) = \cos(\gamma) = 1 \quad (5)$$

Derived from formula (3-5):

$$B_{xo} = B_x$$

$$B_{yo} = B_y + \gamma B_x$$

$$B_{zo} = B_z + \alpha B_y + \beta B_x \quad (6)$$

In summary, the instrument error of the triaxial magnetometer is composed of three parts: triaxial verticality error, scale coefficient error and zero bias error, and the three types of error characteristics are synthesized, and the instrument error model of the actual triaxial magnetometer is finally established as shown in equation (7):

$$B_{xo} = k_1 B_x + e_x$$

$$B_{yo} = k_2 B_y + k_3 \gamma B_x + e_y$$

$$B_{zo} = k_3 B_z + k_3 \alpha B_y + k_3 \beta B_x + e_z \quad (7)$$

By Equation (7), the triaxial magnetometer error model can be simplified to:

$$B_o = AB + E \quad (8)$$

Where A and E are shown as:

$$A = \begin{bmatrix} k_1 & 0 & 0 \\ k_2 & 1 & \gamma \\ k_3 \alpha & 0 & 1 \end{bmatrix} \quad E = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} \quad (9)$$

Equation (8) is derived to:

$$B = A^{-1}(B_o - E) \quad (10)$$

Equation (10) is the magnetometer error correction model, after the correction parameters, you can find the matrix A and E, and then convert the measured value of the sensor into the actual magnetic field value, to achieve the purpose of sensor error correction. Therefore, the problem of error correction of triaxial magnetometer can be transformed into an estimation problem of 9 correction parameters, and particle filtering algorithm is a commonly used method for estimating parameters of dynamic systems.

### 3. Research Methodology

#### 3.1. Particle filtering algorithms

To describe a dynamic system, define the state transition equations and measurement equations for the system as follows:

$$x_k = f_k(x_{k-1}, w_{k-1})$$

$$z_k = h_k(x_k, v_k) \quad (11)$$

Where  $f_k$  and  $h_k$  are the state and output equations,  $z_k$  is the measured value,  $x_k$  is the state variable,  $w_k$  and  $v_k$  are the mutually independent state and measurement noise.

Bayesian estimation theory can be based on the existing quantitative measurements  $z_{1:k} = [z_1, z_2, \dots, z_k]$  and solving for the estimates of the system state  $x_k$  by two processes, prediction and update, or it can be considered as solving for

the posterior probability density  $p(x_k | z_{1:k})$  of  $x_k$ . The process of recursive estimation can be divided into two steps as follows.

(1) Prediction process. The prior probability density of the state variables  $x_k$  is calculated by the following equation:

$$p(x_k | z_{1:k-1}) = \int p(x_k | x_{k-1}) p(x_{k-1} | z_{1:k-1}) dx_{k-1} \quad (12)$$

(2) Update process. Use the measured values  $z_{1:k}$  and the following updated formula to calculate the posterior probability distribution:

$$p(z_k | x_k) p(x_k | z_{1:k-1})$$

$$p(x_k | z_{1:k}) =$$

$$\frac{p(z_k | z_{1:k-1})}{p(z_k | z_{1:k-1})} \quad (13)$$

However, this recursive method for computing posterior probabilities is only a theoretical method and is difficult to compute directly in practical situations, but it is possible to solve for suboptimal solutions by means of particle filtering algorithms.

The basic idea of the particle filtering algorithm is to construct a sample-based posterior probability

$\{x_k^i\}_{i=1}^N$

density function, use the set of  $N$  particles  $x_{0:k}^i, w_{k,i=1}^N$  to represent the posterior probability density function  $p(x_{0:k} | z_{1:k})$ , and based on the above set of particles, the posterior probability density of the moment  $k$  can be approximated as:

$$\frac{1}{N} \sum_{i=1}^N p(x_{0:k} | z_{1:k}) \approx \hat{p}(x_{0:k} | z_{1:k}) = \delta(x_{0:k} - x_{0:k}^i)$$

$$\frac{1}{N} \sum_{i=1}^N \quad (14)$$

Assuming that the set of particle set  $x_{0:k}^i$  obeys a normal distribution, as  $N$  tends to infinity,  $\hat{p}(x_{0:k} | z_{1:k})$  approximates  $p(x_{0:k} | z_{1:k})$ , combined with equation (14), we can get:

$$\hat{p}(x_{0:k} | z_{1:k}) = \frac{1}{N} \sum_{i=1}^N p(x_{0:k}^i | z_{1:k}) = \frac{1}{N} \sum_{i=1}^N p(x_{0:k}^i | z_{1:k-1}) p(z_k | x_{0:k}^i) w_k^i$$

$$E \quad (15)$$

$$\frac{1}{N} \sum_{i=1}^N p(z_{1:k} | x_{0:k}^i) p(x_{0:k}^i | z_{1:k-1}) w_k^i =$$

$$\frac{1}{N} p(z_{1:k} | z_{1:k-1}) \pi(x_{0:k} | z_{1:k}) \quad (16)$$

Since the true posterior distribution of the system is difficult to obtain, a probability density function of a known distribution  $\pi(x_k | z_k)$  is to be used as a substitute. Since the measurements at  $k$  are independent of the measurements at  $k-1$ , by applying Bayes' theorem, we can get:

$$\pi(x_k | z_k) = \pi(x_k | x_{k-1}, z_k) \pi(x_{k-1} | z_{k-1}) \quad (17)$$

Therefore, it is possible to sample in the probability density function  $\pi(x_k | x_{k-1}, z_k)$  and then merge with the previous sample set to form a new sample set, and each particle in the sample set has its weights, called importance weights. A simple but effective sampling method is to sample from the transfer probability density of the state variable, i.e., let  $\pi(x_k | x_{k-1}, z_k) = p(x_k | x_{k-1})$ , then the importance weights are:

$$w_k^i = w_{k-1}^i p(z_k | x_k^i) \quad (18)$$

### 3.2. Fluxgate error correction method based on particle filter

Equation (10) is the magnetometer error correction model, when the three axis magnetometer measures in the area where the geomagnetic field is evenly distributed, the ideal output value is a strictly orthogonal geomagnetic field

tripartite value. When the three axis magnetometer moves in space for attitude transformation, it gets different three component values, but they have the same modulus, which is the principle of "attitude independence" calibration. Using this principle, a systematic model for the error parameter estimation of the three axis magnetometer is established by taking the square of the geomagnetic vector mode measured by the three axis magnetometer as the systematic observation measurement.

The nine correction parameters to be estimated  $k_1, k_2, k_3, \alpha, \beta, \gamma, e_x, e_y, e_z$  are used as the state variables of the system, and the parameters to be estimated remain unchanged during calibration, so the particle filter estimation system's equation of state for the error parameters of the three axis magnetometer is:

$$X(k) = X(k-1) \quad (19)$$

The whole correction process mainly includes three parts: experimental acquisition of measurement data, particle filter system parameter estimation, and error correction. The specific steps of this method are as follows:

- (1) Select the area without external interference magnetic field for experiments, and use the scalar magnetometer and the three-axis vector magnetometer to obtain the measurement value of the surrounding magnetic field;
- (2) Bring the measurement values of the two magnetometers in (1) into the equation of state and the measurement equation of the particle filtering process, iteratively update the particle weights, and obtain the optimal estimate of the system state, which is the parameter estimate of the final correction model;
- (3) Substitute the estimated parameter values and the measured magnetic field data into the error compensation model of Equation (10) to obtain the corrected geomagnetic field vector measurements.

#### **4. Simulation and experiment**

##### **4.1. Simulation**

In order to verify the effectiveness of the proposed correction method, the randomization method is used to generate the three-axis magnetic field data of any attitude in space. Assuming that the geomagnetic field is  $B$ , then under the ideal three axis orthogonal coordinate system  $XYZ$ , the three axis components are:

$$B_x = B \cos(\theta_1) \cos(\theta_2)$$

?

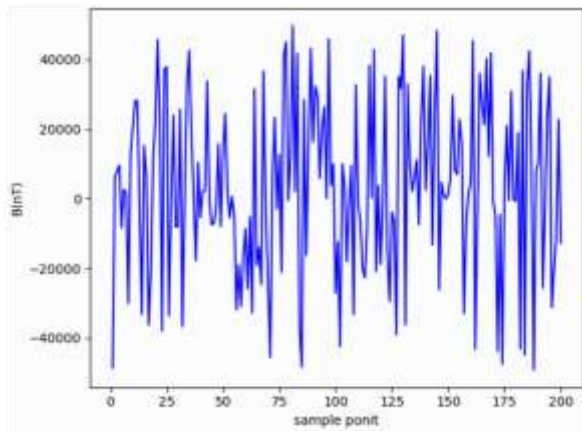
$$B_y = B \cos(\theta_1) \sin(\theta_2)$$

?

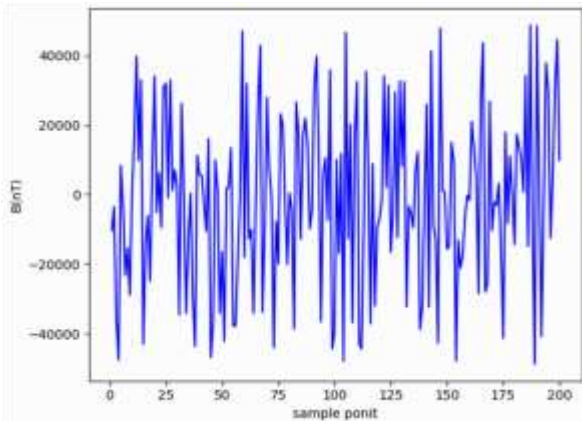
$$B_z = B \sin(\theta_1) \quad (20)$$

Where,  $-\frac{\pi}{2} \leq \theta_1 \leq \frac{\pi}{2}, 0 \leq \theta_2 \leq \frac{\pi}{2}$ .

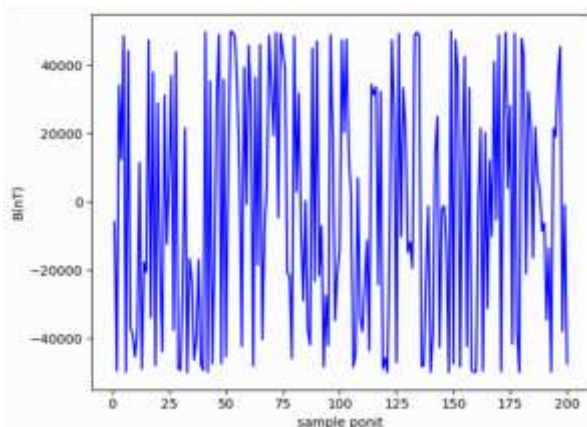
Assuming that the geomagnetic field is 50,000nT, 200 sets of  $\theta_1, \theta_2$  are generated using the randomization method. After substituting into equation (20), we can yield 200 sets of three axis orthogonal magnetic field values at different attitudes, and the resulting three axis magnetic field values are shown in Figure 1-3.



*Figure 1: Magnetometer X-axis output value*



*Figure 2: Magnetometer Y-axis output value*



*Figure 3: Magnetometer Z-axis output value*

The maximum total field of the magnetometer is 51671 nT, the minimum value is 48386 nT, and the change amplitude reaches 3285 nT. As can be seen from Figure, since the shafts of the triaxial flux gate sensor are not orthogonal, the output total field value fluctuates widely when the sensor attitude changes, so the error correction

of the measured value of the sensor is required. 50000nT is used as the target measurement value, and the model parameters are updated and iteratively updated based on the particle filtering algorithm, and the estimation results of each correction parameter are shown in Table 1.

Table 1: Calibration parameter estimation results

|        | zero offset value | sensitivity | Non-orthogonality |
|--------|-------------------|-------------|-------------------|
| X-axis | -70.7557          | 0.9703      | -0.0172           |
| Y-axis | -57.4770          | 1.0126      | 0.0223            |
| Z-axis | 42.5479           | 1.0189      | -0.0253           |

The sensor correction parameter is substituted (9) to calculate the three-axis output value of the modified sensor. Figure 4 shows the comparison of the total field value before and after the sensor correction, and Figure 5 shows the error curve after correction. After correction, the total field output value at any attitude is about 50000nT. The maximum total field value is 50030 nT, the minimum value is 49998 nT, and the change is reduced to 32 nT.

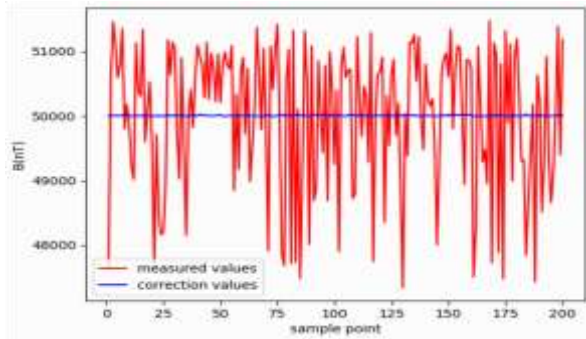


Figure 4: Contrast curve of the total field value after correction

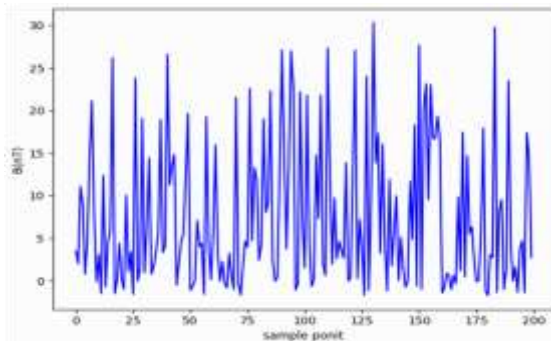


Figure 5: Total field value error curve after correction

#### 4.2. Experimental results and error analysis.

The error correction experiment of a certain type of three axis fluxgate sensor is carried out, and the placement attitude of the flux gate sensor is arbitrarily changed in a relatively stable geomagnetic field environment, and the three axis output of the sensor is tested and the results are recorded, and the three axis magnetic field value is shown in Figure 6-9.

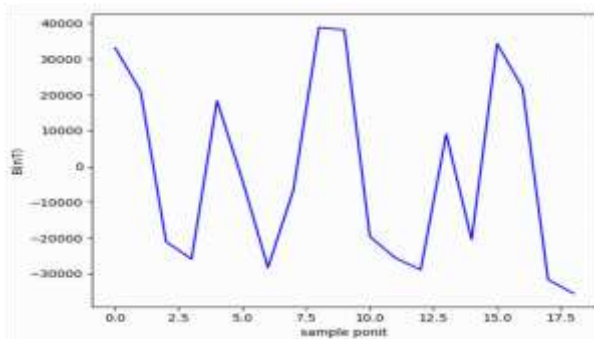


Figure 6: Magnetometer X-axis output value

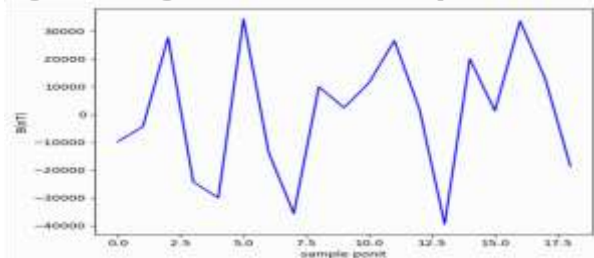


Figure 7: Magnetometer Y-axis output value

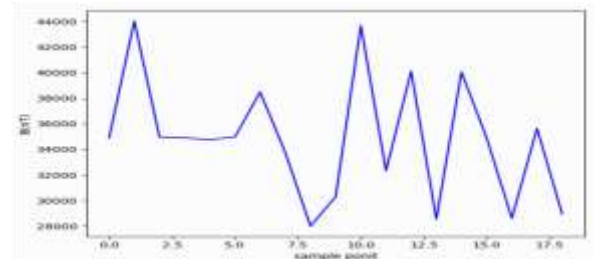
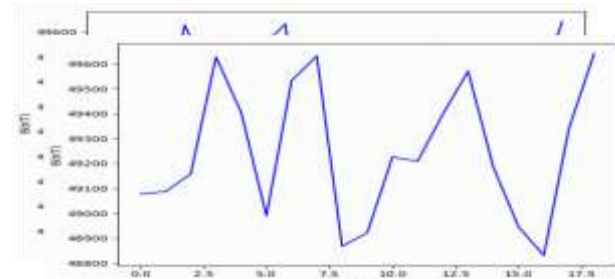


Figure 8: Magnetometer Z-axis output value



The maximum total combined magnetic field is 49642nT, the minimum is 48830nT, and the variation amplitude reaches 812nT. Due to the small fluctuation of the local geomagnetic field, there is a certain error in the magnetic field measured by the uncorrected flux gate sensor, which must be corrected by the error correction algorithm. Based on the particle filtering algorithm, the model parameters are updated and iteratively updated, and the estimated results of each correction parameter are shown in Table 2.

Table 2: Calibration parameter estimation results

|        | zero offset value | sensitivity | Non-orthogonality |
|--------|-------------------|-------------|-------------------|
| X-axis | 156               | 0.9303      | 0.00490           |

|        |     |        |         |
|--------|-----|--------|---------|
| Y-axis | 122 | 1.0126 | 0.00025 |
| Z-axis | 88  | 1.0189 | 0.00092 |

The sensor correction parameter is substituted (10) to calculate the three-axis output value of the modified sensor. Figure 10 shows a comparison of the total field values before and after the sensor correction, and Figure 11 shows the corrected error curve. After correction, the total field change at any pose is reduced to 18nT.

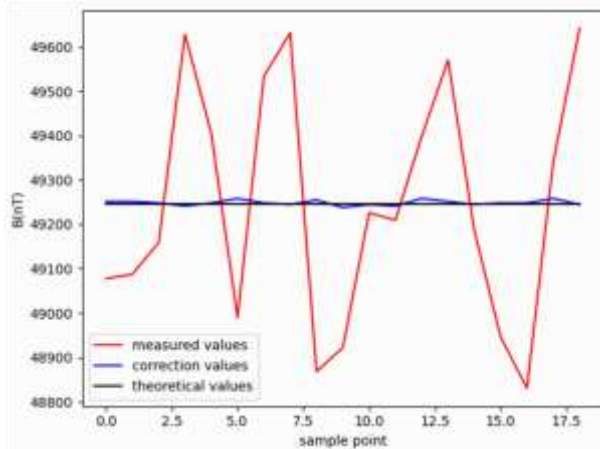


Figure 10: Contrast curve of the total field value after correction

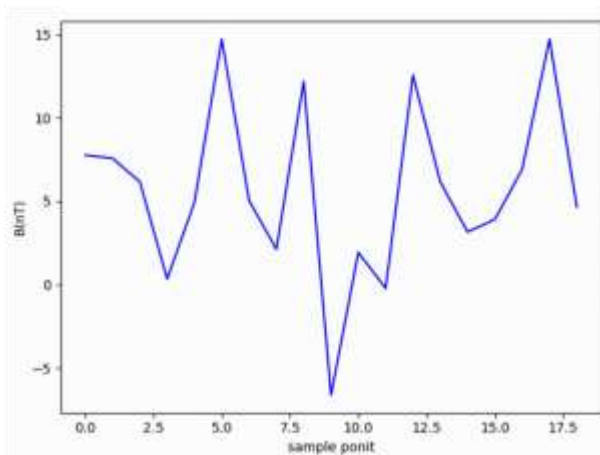


Figure 11: Total field value error curve after correction

The simulation results show that the proposed correction method can correct the sensor errors to a large extent, although there are still some errors.

It can be seen from the experimental results that through the error correction model, the measured ambient magnetic field strength value of the sensor at different attitudes tends to be stable, the oscillation amplitude of the geomagnetic field measurement value is significantly reduced, and the test error caused by the sensor manufacturing process is effectively compensated within a certain range. Through the analysis of the test method and test data, when the error correction experiment is carried out, there is still a certain magnetic interference noise around the sensor, which affects the accuracy of the test data. In addition, the target value in the error correction algorithm has a certain deviation from the real geomagnetic field value, which also affects the accuracy of the correction result. In the subsequent test, the magnetic field value measured by the optical pump can be considered to replace the

calculation, improve the accuracy of the correction, and reduce the magnetic interference of the surrounding environment, so as to establish an ideal error correction test environment and improve the accuracy of error correction.

## **5. Conclusion**

In order to correct the test error caused by the non-orthogonality of the coordinate system of the triaxial flux gate sensor, the inconsistency of the electrical performance of each axis and the zero point drift, an error correction method based on particle filter parameter estimation is proposed. This method can be used to correct the test error caused by the above three reasons at the same time, so that its performance is closer to the ideal triaxial flux gate sensor. The actual geomagnetic field measurement experiment shows that after the error correction of the sensor output by the proposed method, the measurement accuracy of the geomagnetic field of the triaxial flux gate sensor has been significantly improved. Therefore, the error correction method of the triaxial flux gate sensor proposed in this paper is effective, and it can provide a reference for the practical application of fluxgate magnetometer with high accuracy request.

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