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# AMBIGUITY IN RISK MANAGEMENT: A FRESH APPROACH

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***Abstract:***
*Decision making is an integral part of human existence, and the assessment of risk and uncertainty plays a pivotal role in this process. This paper aims to elucidate the subtle distinctions among various related terms, with a primary focus on risk, which directly influences financial and managerial decisions. It is essential to recognize that while some decisions may entail a high degree of certainty, others are riddled with uncertainty, ambiguity, and risk. In the realm of decision making, success or failure hinges on the ability to acknowledge and account for potential risk factors and uncertainties. Despite a substantial body of research in this domain, many studies fail to differentiate between risk and certainty, leading to a muddling of concepts and, at times, erroneous conclusions. This paper addresses this issue, aiming to provide clarity and differentiation among these closely related terms. The distinction between risk and uncertainty is pivotal, as it enhances the precision of decision-making models, preventing theoretical and empirical ambiguities.*
*Through a conceptual approach complemented by mathematical and numerical applications, this paper dissects the various related terms, bringing clarity to the understanding of risk, an essential concept for financial and managerial decisions.*

**Keywords:** Decision making, risk, uncertainty, ambiguity, financial decisions, managerial decisions

**Introduction**
How certain can we be of the nature and direction of the consequences to any decision we make? The rational answer would most likely be that we cannot be one hundred percent sure! but some degree of certainty can be discerned, analyzed and estimated, along with some uncertainty, ambiguity and risk. Some of our decisions are made under the right circumstances that allow for an excellent degree of certainty, while other decisions are made under less fortunate circumstances, allowing different degrees of uncertainty and risky conditions. However, life experience shows that there has always been an atmosphere of uncertainty and risk surrounding all decisions, no matter how well-suited the circumstances, how well-deliberated the process, and how meticulously checked the calculations. Experience has also shown that success and failure can be determined by how potential factors of risk and uncertainty are recognized and accounted for. There has been a considerable amount of research done on this very subject, namely the process of decision making under the condition of risk and uncertainty, but surprisingly the vast majority of the published studies, not only did not distinguish between risk and certainty, but also neglected to recognize the other related concepts and constructs, and therefore underestimate their role in determining the outcome. Groot and Thurik (2018) reported that, “88.3% of articles in this topic does not adhere to the distinction between risk and uncertainty” (p.4)! Not to mention the distinction among other related terms that are assumed to be interchangeable. The authors continue to declare that not distinguishing between these closely related terms would “contribute to the contamination of the concepts that currently dominate the literature and make research prone to confusion, and may lead researchers to erroneous conclusions” (p.5), and undesirable theoretical and empirical consequences. This paper will shed somelight on differentiating all the related terms to risk and uncertainty, and specifically focus on risk, being the core construct directly related to the financial and managerial decisions. The approach is conceptual and supported by mathematical and numerical applications.

## 2. Differentiating the Interrelated Concepts

The most relevant meaning of risk and its related concepts come in the context of economics. It goes back to the American economistFrank Knight (1885-1972) and his 1921study.**Certainty** in this context refers to the condition of having one possible outcome that is known and absolutely confirmed to the decision maker. Contrary to that, and whenever there is a possibility of having more than one outcome, the condition would be considered either risky or uncertain. This would lead us to the distinction between risk and uncertainty, the two terms that may have been used interchangeably all along despite having a thin but crucial line between them, especially in the context of managerial and financial considerations.

**Risk** refers to the condition in which there are multiple possible outcomes where the probability of each alternative outcome is either known or can be estimated. **Uncertainty** shares the first element with risk, the existence of multiple possible outcomes but differs in the second element such that the probability of each outcome is either unknown or cannot be estimated. Uncertainty includes two types:**Total uncertainty** is where conditions are entirely unknown and there is no guidance to their inference; while **Partial uncertainty** leaves the possibility of inference to a set of subjective assumptions. This partial uncertainty is treated similar to the risky conditions in the context of managerial decisions. Differentiating the concepts would imply the distinction between their sub-terms such as risk aversion and uncertainty aversion. **Risk aversion** refers generally to avoiding the unknown, which leads to preferring higher predictability over low predictability of outcomes. **Uncertainty aversion**, which is also sometimes called **Ambiguity aversion** is about the preference of plainly known chance over any unknown chance, even when the reward of the known chance is less than the reward of the unknown chance. This concept can be illustrated by what became known as Ellsberg Paradox. **Uncertainty avoidance** is another term that became specifically associated with social and cultural contexts, where societies and cultures are differentiated based on how tolerating they are to unpredictability in the social and cultural changes. In other words, how comfortable a society or culture is with the unknown, unconfirmed, or unconventional norms, ideas, and practices? Another related but relatively modern term is **Loss aversion**, which has been associated with the 1991 study by Tversky and Kahneman. It refers to the unequitable extent of dissatisfaction/satisfaction related to the loss/gain of an equal monetary sum. In other words, it is about people's general tendency to avoid a loss, even if there is a gain of the same amount to even it out!It is simply because their dissatisfaction with the loss exceeds their satisfaction with the gain of the same amount. This logic is consistent with the economic theory of the diminishing marginal utility of wealth which suggests that a person's utility would drop more if a dollar is lost than it would rise if a dollar is gained.

## 3. Ellsberg Paradox

Following earlier notions of Keynes and others, American economist Daniel Ellsberg popularized this paradox about people's preference of choices with seemingly clear likelihood over choices with less clear likelihood. His experiment involved people's choices of two urns, each of which contains 100 balls. People were told that the first urn contains 50 red balls and 50 black balls, while the second urn contains unknown mix of red and black balls. A reward would be given to anyone who can blindly pull a red ball from one of the urns! People have overwhelmingly chosen the first urn to pull from! Obviously, because they thoughtthat the chance is 50% to get a red ball from the first urn, while it is unknown in the second urn. This illustrates that people dismissed the 99% probability of getting a red ball from the second urn, if there were 99 red ball and only one black ball, which was possible since the urn could contain any mix.

## 4. Sources of Risk

Many possible sources can introduce certain conditions of risk into the decision making process. Most of these sources are external to the firm. We can group the most common sources into three categories

**Economic Sources** are related to the economic environment of a country. The fluctuations in the financial market pose a credible risk to the value of assets in the current and future periods. Such a risk is known as "**market risk**". Major economic factors such as inflation and interest rate pose yet other significant impact on prices and value of lending and borrowing and their impact on earnings. Changes in the credit obligations, and in the state of liquidity can also introduce what are called **credit risk** and **liquidity risk,** in addition to the **currency risk** which can stem from changes in the exchange rate between the domestic and foreign currencies. Also, the state of competition in the same industry or region poses another type of economic risk.

**Political Sources** are related to the government policies, domestically and internationally, that may introduce certain risk on an industry in particular, or on the economy as a whole. Changes in tax policies is a typical example, and **expropriation risk** is another example. This risk arise where a government abroad seizes a property, restricts the rights, or remove the privileges of the hosted firm.

Terrorism and cybercrimes nowadays constitutes a significant political risk on business activities of all firms, domestically and globally.**Social Sources** are related to cultural or religious reasons or to certain social norm or trend that affect consumer preferences and demand. Certain food or clothing items or weather related products may not have any chance to be marketed in certain countries, which is a risk to be accounted for. Even domestically, consumer taste and preferences are subject to change and any business which cannot respond and keep up with those changes would face the risk of being outdated or off-trend and may lose its market share.**International Sources**are related tocommercially or politically competitive reasons among countries.

**5. Measurement of Risk**

As it involves calculable multiple outcomes, risk can be defined in terms of the variability of those outcomes and to what extent they are dispersed. The relationship between risk and variability and dispersion of outcomes is direct. Large variability and wide dispersion would mean higher risk and small variability and tight distribution of outcomes indicates lower risk. For example, if an investment opportunity earns 5% fixed and guaranteed rate of return, and another opportunity may earn anywhere between -10% and 30%, we can easily discern that it would be considered risk free in the first opportunity and highly risky in the second opportunity. Such a realization of the higher risk is definitely based on the wide range of possibilities of the earned return in the second opportunity. Ironically, in considering this example, we can also vividly see that the only possibility of earning a very high return such as 30% would be available only with the risky package, hence the direct relationship between risk and return. Seeking higher return means the willingness to deal with higher risk and seeking security means accepting a modest return. Risk, therefore, can be measured by the classic statistical measurement of dispersion. That is variance or standard deviation. We can classify risk measures into an absolute and relative measure of risk. The objective of the **absolute measure**is to see how the actual outcome is deviating from the expected value. Can we guess how risky some assets by only looking at their returns?Let's contemplate the range of returns for X and Y assets, and take it as a hint to the dispersion of returns, and let's assume that there are three returns for each. We can see, in the following table, that the difference between the highest and lowest return for each would refer to more dispersion for asset Y (range: 15 - 5 = 10), than for asset X (range: 11 - 9 = 2). This may indicate that asset Y is riskier than asset X for having higher variability of returns.

| Return | X | Y |
|---|---|---|
| $k_1$ | 9 | 5 |
| $k_2$ | 11 | 10 |
| $k_3$ | 11 | 15 |
| Range: $k_3 - k_1$ | 2 | 10 |

This simple variability notion can better be represented by the probability distribution of returns. The tighter the probability distribution, the more likely to have the actual return be close to the expected value, and therefore the lower the risk for that asset, and vise versa. The statistical variance ($\sigma^2$) would provide a measure of variability or dispersion for it is the weighted average of the squared deviations from the mean:

$$\sigma^2 = \sum_{i=1}^{n} [x - \bar{x}]^2 P(x) \; ; \; \bar{x} = \text{Expected Value} = \sum_{i=1}^{n} x_i P(x_i)$$

Where $\bar{x}$ is the mean or the expected value of outcomes.Risk as expressed by variability or dispersion of outcomes can also be measured by the standard deviation ($\sigma$) as it is the squared root of variance ($\sigma^2$):

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| Asset | $x_i$ | $P(x_i)$ | $x_i P(x_i)$ | $x_i - \bar{x}$ | $[x_i - \bar{x}]^2$ | $[x_i - \bar{x}]P(x_i)$ |
|---|---|---|---|---|---|---|
| Asset X | 9 | .25 | 2.25 | -1 | 1 | .25 |
| | 10 | .50 | 5.0 | 0 | 0 | 0 |
| | 11 | .25 | 2.75 | 1 | 1 | .25 |
| Asset Y | $\bar{x} = \sum x_i P(x_i) = 10$ | | | $\sigma^2 = \sum [x_i - \bar{x}]^2 P(x_i) = .5$ | | |
| | 5 | .25 | 1.25 | -5 | 25 | 6.25 |
| | 10 | .50 | 5 | 0 | 0 | 0 |
| | 15 | .25 | 3.75 | 5 | 25 | 6.25 |
| | $\bar{x} = \sum x_i P(x_i) = 10$ | | | $\sigma^2 = \sum [x_i - \bar{x}]^2 P(x_i) = 12.5$ | | |

$\sigma_x = \sqrt{.5} = .71$; $\sigma_y = \sqrt{12.5} = 3.5$

$$\sigma = \sqrt{\sum_{i=1}^{n} [x_i - \bar{x}]^2 P(x_i)}$$

If we assume the probabilities of the returns to X and Y assets are 25%, 50%, and 25% respectively, we can calculate the standard deviations for the three returns.

The standard deviation of .71 means that the returns on asset X are much closer to their own expected value than the returns on asset Y which has a standard deviation of 3.5, indicating how wide the dispersion of returns.

In the long run, asset risk would be an increasing function of time. Project I

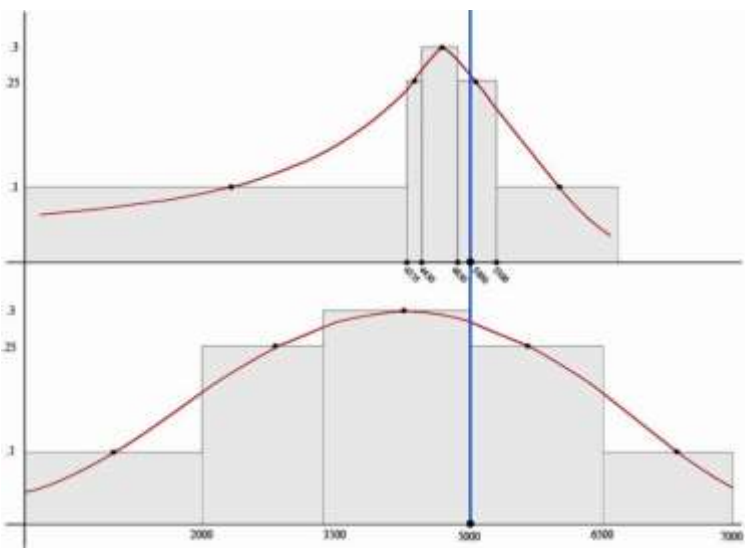
| Years | Predicted Profits $x_i$ | Probability $P(x_i)$ | $x_i P(x_i)$ | $[x_i - \bar{x}]$ | $[x_i - \bar{x}]^2$ | $[x_i - \bar{x}]^2 P(x_i)$ |
|---|---|---|---|---|---|---|
| 1 | 4,375 | .10 | 437.5 | -625 | 390,625 | 3,9062.5 |
| 2 | 4,450 | .25 | 1,112.5 | -550 | 302,500 | 75,625 |
| 3 | 4,850 | .30 | 1,455 | -150 | 22,500 | 6,750 |
| 4 | 5,300 | .25 | 1,325 | 300 | 90,000 | 22,500 |
| 5 | 6,700 | .10 | 670 | 1,700 | 2,890,000 | 289,000 |
| $\bar{x} = \sum_{i=1}^{5} x_i P(x_i) = 5{,}000$ | | | | $\sigma^2 = \sum_{i=1}^{5} [x_i - \bar{x}]^2 P(x_i) = 432{,}937$<br>$\sigma =$ [redacted] $= 658$ | | |

Project II

| Years | Predicted Profits $x_i$ | Probability $P(x_i)$ | $xP(x_i)$ | $[x_i - \bar{x}]$ | $[x_i - \bar{x}]^2$ | $[x_i - \bar{x}]^2 P(x_i)$ |
|---|---|---|---|---|---|---|
| 1 | 2,000 | .10 | 200 | -3,000 | 9,000,000 | 900,000 |
| 2 | 3,500 | .25 | 875 | -1,500 | 2,250,000 | 562,500 |
| 3 | 5,000 | .30 | 1,500 | 0 | 0 | 0 |
| 4 | 6,500 | .25 | 1,625 | 1,500 | 2,250,000 | 562,500 |
| 5 | 8,000 | .10 | 800 | 3,000 | 9,000,000 | 900,000 |

| | |
|---|---|
| $\bar{x} = \sum_{i=1}^{5} x_i P(x_i) = 5,000$ | $\sigma^2 = \sum_{i=1}^{5} [x_i - \bar{x}]^2 P(x_i) = 2,925,000$ <br> $\sigma =$ [redacted] $= 1,710$ |

The variability of returns gets wider and the risk gets greater as time goes by. Practically, this would be translated as that the longer the life of an investment asset, the higher the risk involved. Suppose that two investment proposals were submitted to a firm for funding, with their own estimations of the profits (in hundreds of thousands of dollars) in the next five years. We can expect that the financial advisors/managers would make their assessment and choice based on some measures such as the one described above.




Since the two projects will yield the same expected value, the next crucial criterion would be which of them is safer or riskier than the other. The answer would be clear at the calculation of variance ($\sigma^2$) and standard deviation ($\sigma$). The calculated results show that Project I had a smaller standard deviation (658) than Project II (1,710). Project I would win for being less risky than Project II. It is clear on the graph how data of Project II are dispersed over horizontally, forming a widely spread curve while they are much tighter in Project I, which shows how the outcomes are generally close to the expected value or mean.

Assuming the distribution is normal, it would mean that:

There is a 68.26% chance that the actual outcome is within one standard deviation from the expected value. Based on the symmetry of the normal distribution, this chance is divided equally between a negative 34.13% and a positive 34.13%. So, if the standard deviation is 1,710, for example, there would be a 34.13% chance that the actual value is 5,000 + 1,710, and a 34.13% chance that it is 5,000 - 1,710. So the general range would be from 3,290 to 6,710. The chance would increase to 95.44% within two standard deviations (2 x 1,710), which is also split equally on both sides of the mean. In this case, the chance would be 47.7% that the range of the actual outcome would be between 1,580: [5,000 - (2 x 1,710)] and 8,420: [5,000 + (2 x 1,710)].

It is clear that if we deal with a smaller standard deviation such as the 658, the ranges of the actual outcome would be closer to the expected value, rendering more security and less risk.This interpretation is, of course, not limited to the discrete one, two, or three standard deviations from the mean. it would apply to any range in between. Therefore, we can find the probability of a specific outcome ($x_i$) such as 5,500, for example. We can calculate how much of a standard deviation from the mean($\bar{x}$) this value would reveal by calculating the value of Z and looking up the statistical table of the normal distribution Z = $x_i$ ? $\bar{x}$; Z = 5,500?5,000 ? .76

? 658

This means that if we have an actual outcome of 5,500, it would fall within a little more than three quarters of a standard deviation from the expected value. Looking at the table between .7 and .8 vertically (zvalue) and under 2% horizontally, we can see that the area under the curve would be between .26 and .29.

Thesecond type is the**Relative Measure,** which is helpful whenwe have projects with different expected values. It requires that the standard deviations has to be relative to their expected values, hence the calculation of the coefficient of variation (V) which measures the outcome dispersion as it is related to each of

?

the expected values individually, V ? $\bar{x}$. In this sense, the measure of risk would be translated into a measure of standard deviation per unit of the expected value. The relationship between the coefficient of variation and risk is still as positive. So, the criteria would be "the lower the value of the coefficient the lower the risk, and vice versa. In our absolute measure in the last example, Project II (σ = 1,710) was riskier than Project I (σ = 658) while both would yield the same expected value ($\bar{x}$ ?5,000). Suppose now that Project II has an expected value of $6,000. It would still be riskier than Project I if we compare their coefficient of variation (V):

VI ??I ? 658 ?.13 ; VII ??II ? 1,710 ?.28 $\bar{x}_I$ 5,000 $\bar{x}_{II}$ 6,000

where Project II revealed a higher coefficient of variation reflecting a higher risk.

## 6. Risk Aversion

Following what was said before, risk aversion can be translated into people's general tendency to avoid, or at least minimize, all sorts of risk and uncertainty when they make decisions. Decision makers are described based on three major attitudes towards risk. A **Risk Averter**who wouldchoose no risk , or at best choosesthe lowest possible level of risk. A **Risk Taker**who prefers to venture and gets involved in risky situations and in conditions that require a higher level of speculation in pursuit of the highest possible payoff. A **Risk Neutral**who is indifferent to risk, and only focuses on expected returns much more than to pay any attention to the way those returns are dispersed. Although it has been very well established in the business world that the highest return is usually associated with the highest risk, most people, and specifically managers, are naturally risk averters, especially when larger potential losses are involved. It has been observed that even risk neutrals would turn into risk averters when large amounts of money are at stake.

Suppose that a group of people in a club decided to play a coin gamble, and since many wanted to play, the following rules were put forward:If a head turns up, the player wins $200. If a tail turns up, the player loses $100, and because of the competition to play, $10 is offered to the player who gives up his turn, or basically pledges not to play. According to the aforementioned attitudes towards risk, a risk averter would have no problem leaving the game and take the $10 for doing nothing. For him, it would be an easy gain, although it is at the expense of foregoing a possibility of gaining $200. For a risk neutral, the focus would be on the weighted average that would come out of this game. He would make his decision based on the fact that in reality there would be an average gain since the amount for gain is higher than the amount for loss while both stand the same probability (50%). In this case, the risk neutral would calculate the expected value:

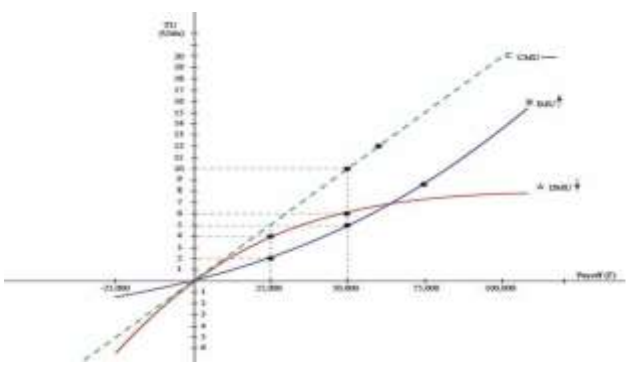
$x_1(p_1) + x_2(p_2) = \bar{x}$; (200)(.5) + (-100)(.5) = 50

A risk taker would be the most enthusiastic to play, focusing on the highest win and he may not hesitate to play again in pursuit of that $200.

**7. Risk Attitudes and Utility of Money**

Risk attitudes can be explained by the utility of the earned or lost money. Each attitude can be represented more accurately by the change in total utility or what we call the marginal utility. Marginal utility is generally decreasing for the risk averter, increasing for the risk taker, and constant for the risk neutral. The following tables contain data on five possible payoffs and both total utility and marginal utility derived from them as subjectively determined by the three types of decision makers.

| Payoff | Decreasing | | Increasing | | Constant | |
|---|---|---|---|---|---|---|
| | Utility | Change | Utility | Change | Utility | Change |
| -25,000 | -6 | / | -1.75 | / | -5 | / |
| 0 | 0 | 6 | 0 | 1.75 | 0 | 5 |
| 25,000 | 4 | 4 | 2 | 2 | 5 | 5 |
| 50,000 | 6 | 2 | 5 | 3 | 10 | 5 |
| 75,000 | 7.5 | 1.5 | 8.5 | 3.5 | 15 | 5 |
| 100,000 | 8.25 | 0.75 | 13.25 | 4.75 | 20 | 5 |



In the following Figure, We can visually observe how three managers represented by their marginal utility curves A, B, and C perceive their monies and assess their utilities. If we take, for example, a payoff of $50,000, it would provide a total utility of 6 utils for the risk averter represented by the diminishing marginal utility concave curve (A); provides 5 utils for the risk taker represented by the increasing marginal utility convex curve (B); and provides 10 utils for the risk neutral manager represented by the constant marginal utility straight line (C). We can observe further differences among these three managers based on their attitudes towards winning and losing money. If, for example, we consider an event that would increase the payoff from $50,000 to $75,000!, how would these managers respond? The risk averter's total utility would increase from 6 to 7.5 gaining 1.5 utils; the risk taker's utility would increase from 5 to 8.5 gaining 3.5 utils; and the risk neutral's utility would increase from 10 to 15, gaining 5 utils. The risk averter ultimately gained the least among the three from the same amount of money. Such a gain would get even less when we move them further in the payoff from $75,000 to $100,000. The risk averter would gain only .75 utils as compared to 4.75 utils for the risk taker and 5 utils for the risk neutral. What this theoretical approximation means is that only the risk neutral would respond proportionally to the change in the monetary payoff. He would take it dollar for dollar or dollar gained would equal dollar lost for him. It is an attitude of indifference towards risk. The table below shows thatsituation, where the percentage change in the last column is identical in both cases with the percentage change in the payoff amounts in the second column. It also shows that the risk averter's utility responds in less than proportional as compared to the change in the payoff where the third column shows .25 and .10 are less than their corresponding changes in the payoff amount, .50 and .30 respectively in the second column. As for the risk taker's utility, it responds in more than proportional to the monetary change in the payoff amount. Notice the percentage changes in the sixth column (.70 and .56) and compare them to their corresponding charges of the payoff amounts in the second column (.50 and .30) respectively. All the percentage changes were obtained by dividing the difference between the later and earlier amounts by the earlier amount %x

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| Payoff | % Δ in Payoff | Risk Averter's Utility | % Δ | Risk Taker's Utility | % Δ | Risk Neutral Utility | % Δ |
| $50,000 | | 6 | | 5 | | 10 | |
| $75,000 | .50 | 7.5 | .25 | 8.5 | .70 | 15 | .50 |
| $100,000 | .30 | 8.25 | .10 | 13.25 | .56 | 20 | .30 |

**8. Expected Utility of Money vs. Expected Monetary Return**

Suppose a manager wants to invest in oil drilling, and he would face the following possibilities: 1) If no oil turns up, he would lose all the investment of $25,000. The probability of this outcome is 80%. 2) If oil is found, the payoff would be $100,000, but this outcome is probable at only 20%.The expected value of investment in oil drilling would be:(100,000)(.20) + (-25,000)(.80) = 0

Obviously, if he decides not to drill, the expected value would be zero already. So, under this fair game, it would not matter what decision the manager makes since both, to drill and not to drill, would eventually lead to a zero outcome. In this scenario, a decision based on the expected monetary value of the payoff would not help. What would help here is to make a decision based on the **utility of money**. Here we would see three different assessments corresponding to three different risk attitudes, as we have seen earlier.Using the previous table of utility, we can calculate the expected value of money utility for the three managers with three attitudes:

1) For the risk averter manager, the expected value of utilityEVmvwould be equal to:

$$EVmv= (Ut_1)(P_1) + (Ut_2)(P_2) = (8.25)(.20) + (-6)(.80) = -3.15.$$

2) For the risk taker: EVmv = (13.25)(.20) + (-1.75)(.80) = 1.25

3) For the risk neutral: EVmv= (20)(.20) + (-5)(.80) = 0

Therefore, it is expected that the risk averter manager would decide not to drill because of the negative expected utility (-3.15) as compared to the expected utility of zero in no drilling. On the contrary, the risk taker would decide to drill based on his positive expected utility of (1.25). The risk neutral manager ended up with no expected utility and therefore it is as good as no drilling.

If we know the function of the utility of money U, such as $U = 400\ m^{.25}$

And we know the initial amount of money (m), then we can test whether the function is increasing or decreasing. This one is an increasing function of money since the first derivative is positive:

dU .75

(.25)(400)m = 100 m.75 > 0 dm

And it is positive for any amount of money larger than zero. The second derivative of the function would determine the type of function for the marginal utility:

$d^2U$

- If it is constant marginal utility, dm 2 0 , it would indicate the a neutral risk attitude.

$d^2U$

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- If it is decreasing marginal utility, $dm^2$ ▯ 0 , it would refer to the risk aversion attitude.

$d^2U$

- If it is increasing marginal utility, $dm^2$ ▯ 0 , it would be the risk taking attitude.

Therefore, the second derivative for our function is:

$d^2U$

$dm^2$ ▯ $(-.75)(100)(m)^{-1.75}$, and if m = \$1,000, then $d^2U$

$dm^2$ ▯ $(-.75)(100)(1,000)^{-1.75} = -4.22$

which confirms that the marginal utility function is decreasing, and the decision maker would be described as a risk averter.Now, let's consider the impact of winning \$500 as well as losing \$500 on an initial amount of money of \$1,000 (m = 1,000).$U_1 = 400\ m^{.25} = 400(1,000)^{.25} = 2,249.36$

If the person wins \$500, m would be 1,500:$U_2 = 400(1,500)^{.25} = 2,489.33$, and if the person loses \$500, m would be 500:$U_3 = 400(500)^{.25} = 1,891.48$, $\Delta U_{1\text{-}2} = U_2 - U_1 = 2,.33 - 2,249.36 = 239.97$

$$\Delta U_{1\text{-}3} = U_3 - U_1 = 1,891.48 - 2,249.36 = -357.88$$

If this game is a coin flipping game, the expected value of utility would be obtained by: $E(U) = \sum \Delta U P_i = \Delta U_1 (P_1) + U_2 (P_2) = (239.97)(.5) + (-357.88)(.5) = -58.95$

Since the expected value of utility turns out to be negative, the decision would be not to get into this gamble.

**9. Risk Discount and Certainty Equivalent**

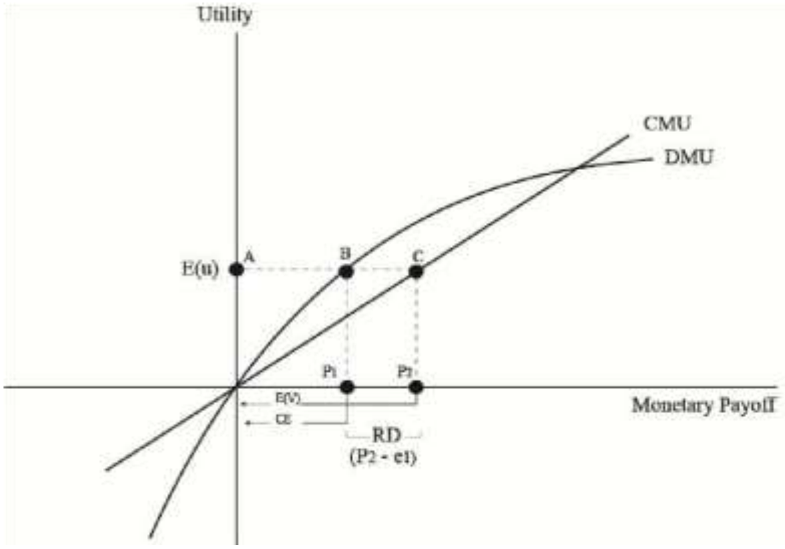
The person who received \$10 as a reward for giving up the aforementioned gamble is certainly a risk averter. This amount of \$10 is called **certainty equivalent (CE)**. It is defined as the compensation which renders the player indifferent to a risky gamble. In that scenario, the expected value of the game was \$50, as the risk neutral player has considered it.

The person who accepted a significantly less outcome (\$10) is for sure a player with a definite risk aversion attitude. This would further define the **risk averter** as one whose certainty equivalent limit is less than the expected value of a certain risk. The difference between the expected value E(v) and the certainty equivalent (CE) is called the **Risk Discount** (RD): RD▯E(v)▯CE

Risk discount shows the extent to which the expected value for a given risk is reduced in order to avoid such a risky prospect. In our previous example, risk discount was \$40:

$$RD = E(v) - CE = 50 - 10 = 40$$

The following figure shows the certainty equivalent and risk discount as we recall the shapes of the curves for the risk averter (the diminishing marginal utility curve DMU), and for the risk neutral (the constant marginal utility curve CMU).



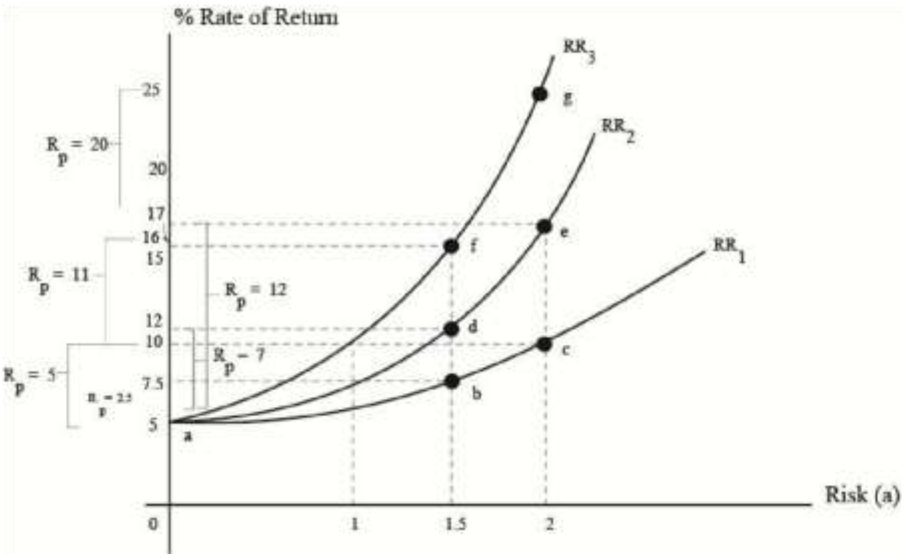

Point A represents the expected utility of the game, the level that would generate two points, B and C, on the risk averter and risk neutral curves, DMU and CMU respectively. From those points, we can drop verticals to see the amounts of payoff for both players. For the risk averter, point $P_1$ would represent the certainty equivalent, and for the risk neutral, point $P_2$ would represent the expected value. The difference ($P_2$ $P_1$) would be the risk discount RD.

**10. Risk Impact on the Valuation Model**

When a firm wants to evaluate the worthiness of an investment project, risk factor should be among the priorities to be considered, as it affects the actual net present value of the project. There are two common ways to adjust the valuation model for risk:

**10.1. Risk Premium Adjustment**

**Risk premium** is defined as the difference between the expected rate of return on a risky investment and the risk-free rate. Let's consider three managers or financial advisors with three different attitudes about risk, as represented by the three curves on the following graph, where risk is on the x-axis, as it is measured by the standard deviation (σ), and rate of return is on the vertical axis. Let's think of these curves in a way similar to the indifference curves. They depict the tradeoff between risk and return from three different attitudes towards risk. The first, RR, represents the least risk-averse among the three. The top, $RR_3$, represents the most risk-averse, and $RR_2$ stands in between.




Point a is on all curves and it shows a 5% risk-free rate (risk = 0). $RR_1$ shows a manager who is indifferent between accepting a 5% rate with no risk or taking a 1.5 σ risk to get a 7.5% return. In other words, for the added risk (from 0 to 1.5σ), his risk premium becomes 2.5% (7.5% - 5%). For the more risk-averse manager on $RR_3$, the move to accept the additional risk of 1.5σ would not be satisfactory unless there is a higher risk premium of 11% so that the required rate of return becomes 16%. Not only that, but if the next opportunity happens to come with an additional risk of .5σ (such as moving from 1.5σ to 2σ), the risk-averse manager on $RR_3$ would want his return to be as high as 25% where his risk premium goes to 20% (25% - 5%). The same level of risk (2σ) would make the manager of $RR_1$ happy to accept only 10% return making his risk premium 5% this time (10% - 5%). As for the moderate manager on $RR_2$, he would accept moderate levels of risk for reasonable rates of return. He would be indifferent between risk-free rate of 5% and 10% rate with 1.5σ risk or 17% rate with 2σ risk. His risk premium would be 7% (12% - 5%) at point d, and 12% (17% - 5%) at point e. The following table shows a comparison between the three positions on risk and return, and risk premium.

The different attitudes by managers towards risk would produce various risk premiums and that would be reflected on the valuation model of a firm as the risk-adjusted rate (k) would replace the risk-free rate (r) that is normally used in the evaluation model: $V = \sum_{t=1}^{n} \frac{\pi_i}{(1+r)^t}$ where V is the value of the asset, $\pi_i$ is the expected profit per year, r is the risk free rate of return so that thevalue is equal to the present value of the future returns or cash flow.

| Risk Attitude | Point | Rate of Return % | Risk Level (σ) | Risk Premium % |
|---|---|---|---|---|
| $RR_1$<br>least risk-averse | a a | 5<br>7.5 | 0<br>1.5 | 2.5 |
| | c | 10 | 2 | 5 |
| $RR_2$<br>moderate risk-averse | a d e | 5<br>12<br>17 | 0<br>1.5<br>2 | 7<br>12 |

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| | | | | |
|---|---|---|---|---|
| $RR_3$<br>most risk-averse | a p g | 5<br>16<br>25 | 0<br>1.5<br>2 | <br>11<br>20 |

When the firm faces the prospect of a risky project, the valuation would be adjusted to the expected risk by incorporating the firm's risk premium ($R_p$). In this case, the net present value NPV of the project would be:

$$NPV = \sum_{t=1}^{n} \frac{i_t}{(1+k)^t} - C_0$$

where k is the risk-adjusted rate of return, which is equal to the risk-free rate (r) used previously, plus the firm's risk premium $R_p$:$k = r + R_p$ , and $C_0$ is the initial cost of the project. The criteria would remain such that an investment is worthwhile when the net present value (NPV) is either equal or larger than zero.$NPV \geq 0$

Suppose a managerial/financial team has to decide on capital allocation for two proposed investment projects, each of which will yield profits for the next 5 years as shown in the following table. They require initial investments of $420,000 and $500,000 respectively. Although the firm's cost of capital is 6%, further investigation revealed certain risk elements associated with both projects.

| Time | Project A cash inflows | Project B cash inflows |
|---|---|---|
| Year 1 | 126,000 | 280,000 |
| 2 | 126,000 | 108,000 |
| 3 | 112,000 | 90,000 |
| 4 | 98,000 | 80,000 |
| 5 | 84,000 | 70,000 |
| Initial Investment | 420,000 | 500,000 |
| r | 6% | 6% |
| $R_p$ | 2½% | 3½% |

The decision makers found it necessary to adjust for risk by assigning risk premiums of 2½ % and 3½ % to both projects respectively. The classic criterion for granting an investment has to utilize calculating the net present value using the risk-adjusted rate of return: First we calculate the net present value of the cash inflows for both projects at the time of their yields using the firm's interest rate (r - 6%). Then we calculate the same net present values, using the risk-adjusted rate (k):

$$k = r + R_p = .06 + .025 = .085 \quad \text{for Project A}$$
$$= .06 + .035 = .095 \quad \text{for Project B}$$

$$NPVA = \sum_{t=1}^{5} \frac{i_t}{(1+r)^t} - C0$$

$$= \frac{i_1}{(1+r)^1} + \frac{i_2}{(1+r)^2} + \frac{i_3}{(1+r)^3} + \frac{i_4}{(1+r)^4} + \frac{i_5}{(1+r)^5} - C^0$$

$$= \frac{126,000}{(1+.06)^1} + \frac{126,000}{(1+.06)^2} + \frac{112,000}{(1+.06)^3} + \frac{98,000}{(1+.06)^4} + \frac{84,000}{(1+.06)^5} - (420,000)$$

= 118,868 + 112,140 + 94,037 + 77,625 + 62,767 - 420,000

= 465,440 - 420,00 = $\boxed{45,440}$

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$$NPV_B = \frac{280,000}{(1+.06)^1} + \frac{108,000}{(1+.06)^2} + \frac{90,000}{(1+.06)^3} + \frac{80,000}{(1+.06)^4} + \frac{70,000}{(1+.06)^5} - (500,000)$$

= 264,151 + 96,120 + 75,565 +63,367 + 52,308 - 500,000

= 551,511 - 500,000 = $\boxed{51,511}$

$$NPV_A{}^{adj} = \frac{126,000}{(1+.085)^1} + \frac{126,000}{(1+.085)^2} + \frac{112,000}{(1+.085)^3} + \frac{98,000}{(1+.085)^4} + \frac{84,000}{(1+.085)^5} - (420,000)$$

= 116,129 + 107,031 + 87,685 + 70,714 + 55,864 - 420,000 = $\boxed{17,423}$

$$NPV_B{}^{adj} = \frac{280,000}{(1+.095)^1} + \frac{108,000}{(1+.095)^2} + \frac{90,000}{(1+.095)^3} + \frac{80,000}{(1+.095)^4} + \frac{70,000}{(1+.095)^5} - (500,000)$$

= 255,707 + 90,073 + 68,549 + 55,646 + 44,466 - (500,000)

= 514,440 - 500,000= $\boxed{14,440}$

At the normal interest rate of 6%, Project B would win the approval of the financial/managerial team since its net present value ($51,511), is larger than that of Project A ($45,440). However, after considering the expected risk involved in both projects, the decision makers would give its approval to Project A due to its larger adjusted net present value of ($17,423) as compared to that of Project B ($14,440).

**10.2. Certainty-Equivalent Adjustment**

As it was explained before, certainty equivalent (CE) is the sure sum that is equal to the expected value E(v) of the risky project. The equivalency is in the utilities of both to the manager or investor, and not necessarily in their monetary values. Let's assume there is a proposal that requires the company to invest $30,000 in a project, where the probabilities of its success and failure are 50/50 between earning $100,000 and earning nothing, respectively. The expected value for such a project would be:

E(v) = 100,000(.5) + 0(.50) = 50,000

If the company approves the funding, it would mean that it is trading off the certainty of $30,000 for a risky expected return of $50,000. In fact, it means that a sure risk-free capital of $30,000 is yielding the same utility of a risky $50,000, hence, the term certainty equivalent to the amount of $30,000 that would make the decision maker indifferent between the two prospects. The certainty equivalent coefficient (α) is the ratio

CE

between the certainty equivalent (CE) and its expected risky return E(v).

E(v)

The certainty equivalent is subjectively determined by the decision maker and, therefore, it would be a product of how risk averse or risk taker is that decision maker. The following graph shows that three different attitudes towards risk would produce three certainty equivalent amounts for the same expected value of $1,000 for a specific risky project: The most risk-averse manager on $RR_3$ would assign $870, the least risk-averse would assign $220, and the moderate manager among the three would assign $460. These cases would produce three different certainty equivalent coefficients:

$$\alpha_3 = \frac{870}{1,000} = .87; \quad \alpha_2 = \frac{460}{1,000} = .46; \quad \alpha_1 = \frac{220}{1,000} = .22$$

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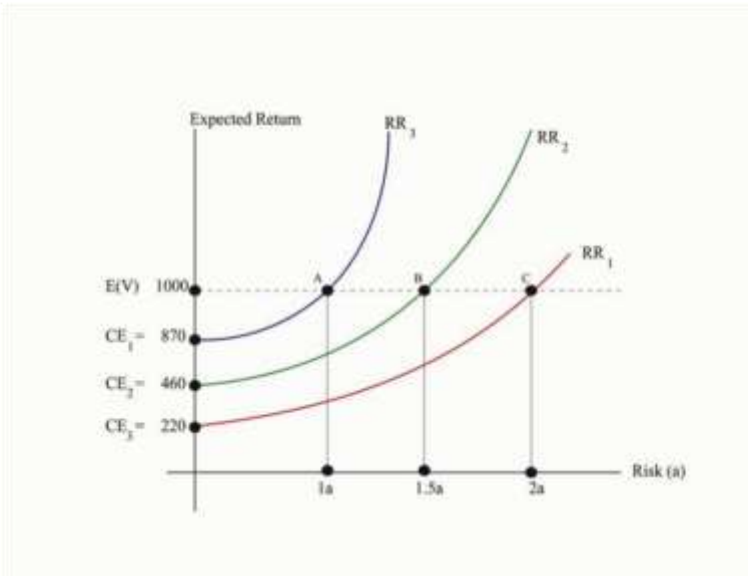


The risk-averse manager would value the sure risk-free money more. That is why his alpha is higher. An alpha of .87 means that each dollar of the certain money would be worth 87¢, as compared to the 22¢ for the risk taker. This is one reason to see why the risk taker dares to take a high risk. Alternatively, each dollar of the expected risky return is valued less ($1.15) for the risk-averse than for the risk taker who values his expected dollar at $4.55.Generally speaking, the criteria for alpha is as follows:

- When α = 0: It is an indication that the probability of getting the expected return does not exist, and therefore the project is too risky to be pursued.
- When α = 1: It refers to the equality between the certainty equivalent (CE) and the expected value of return of the risky project. When the manager or investor gets his return equal to what he assigns as a certainty equivalent, the project is considered risk free.
- When 0<α <1: It is an indication that there is some level of risk. The project is riskier asα value is closer to zero, and less risky if it is closer to 1. It would depend on how smaller the certainty equivalent (CE) is, as compared to the expected value of the risky return E(v).

The valuation model would be adjusted for risk by introducing α to the numerator of the formula as a multiplier to the expected return or profit or cash flow, while the bottom of the formula would keep the risk-free rate (r):

n
NPV ⧠⧠(1⧠⧠⧠r) i t ⧠C0
t⧠1

Let's assume that the manager assigned certainty equivalent sums to each and every annual return of the five years in both projects of the last example. The following table shows α values as it is calculated by dividing the assigned certainty equivalent by the corresponding expected return.

| | A | | | B | | |
|---|---|---|---|---|---|---|
| Time | Expected Return | Certainty Equivalent | α | Expected Return | Certainty Equivalent | α |
| Year 1 | 126,000 | 123,000 | .98 | 280,000 | 240,000 | .86 |
| 2 | 126,000 | 123,000 | .98 | 108,000 | 100,000 | .93 |
| 3 | 112,000 | 106,000 | .95 | 90,000 | 86,000 | .95 |
| 4 | 98,000 | 95,000 | .97 | 80,000 | 76,000 | .95 |
| 5 | 84,000 | 82,000 | .98 | 70,000 | 68,000 | .97 |

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| | | |
|---|---|---|
| Initial investment | 420,000 | 500,000 |
| r | 6% | 6% |

Applying those calculated alphas, we get:
n
NPVA ⎕⎕t⎕1 (1⎕⎕⎕r) i t ⎕C0

$$= \frac{(.98)(126,000)}{(1+.06)^1} \quad \frac{(.98)(126,000)}{(1+.06)^2} \quad \frac{(.95)(112,000)}{(1+.06)^3} \quad \frac{(.97)(98,000)}{(1+.06)^4} \quad \frac{(.98)(84,000)}{(1+.06)^5} ⎕ ⎕ ⎕ ⎕$$

⎕420,000

= (116,490 + 109,897 + 89,335 + 75,296 + 61,514) - 420,000

= 452,532 - 420,000 = 32,532

$$NPv_B = \frac{(.86)(280,000)}{(1+.06)^1} + \frac{(.93)(108,000)}{(1+.06)^2} + \frac{(.95)(90,000)}{(1+.06)^3} + \frac{(.95)(80,000)}{(1+.06)^4} + \frac{(.97)(70,000)}{(1+.06)^5} - 500,000$$

= (227,170 + 89,391 + 71,787 + 60,199 + 50,739) - 500,000 = 499,286 - 500,000= -714

Considering the expected risk for both projects in terms of estimating the certainty equivalent and calculating α for each return in every year revealed that Project A is more worthwhile for yielding a positive value of $32,532 while Project B went into a negative net value.

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