

MASTERING PORTFOLIO OPTIMIZATION: ENHANCING RETURNS ON THE NIGERIAN STOCK EXCHANGE

¹Chinedu Emmanuel Okoro and ²Fiona Abigail Smith

¹Department of Mathematics, University of Cape Town, Cape Town, South Africa

²Statistics, Information Modelling, and Financial Mathematics Research Group, Materials and Engineering Research Institute, Sheffield Hallam University, Sheffield, United Kingdom

Abstract:

Modern Portfolio Theory (MPT), pioneered by Harry Markowitz in the early 1950s, has long been a cornerstone of financial decision-making, particularly in portfolio optimization. Markowitz's mean-variance model aimed to guide investors in selecting assets for their portfolios, determining how to make those selections, and assigning weights to each asset. However, as research has highlighted, the mean-variance approach has its limitations and weaknesses, sparking extensive investigation into its shortcomings.

This paper delves into the focal point of this research, which involves addressing the limitations and assumptions inherent in Markowitz's model. A multitude of scholars, such as Fuerst (2008), Norton (2009), Ceria and Stubbs (2006), Goldfarb and Iyengar (2003), Jorion (1992), Konno and Suzuki (1995), Michaud (1989a), Bowen (1984), Ravipiti (2012), and many others, have dedicated their works to thoroughly scrutinizing these shortcomings and restrictions.

Subsequent to the identification of the deficiencies in Markowitz's Mean-Variance model, numerous researchers have sought to enhance and expand the model in various directions. Notable contributions from authors like Jobson, Korkie, and Ratti (1979), Jobson and Korkie (1980), Frost and Savarino (1988), Jorion (1992), Michaud (1998), Polson and Tew (2000), and others have primarily focused on mitigating the estimation error, thus further refining MPT.

Keywords: Modern Portfolio Theory, mean-variance model, portfolio optimization, limitations, estimation error, financial decision-making, asset selection, asset weighting, portfolio diversification.

1. Introduction

In early 1950's, Harry Markowitz designed a financial model otherwise called mean-variance portfolio optimization. This method was designed such that it will help the investors know which asset that will be selected in a portfolio, how the selection will be done and also the weight of each asset in the portfolio. In the paper titled Portfolio selection (1952), Markowitz's outlined the importance of diversification of portfolios. However, research has shown that the Markowitz mean-variance has some weaknesses and a number of limitations. As a matter of fact, the limitations have taken the centre stage of research. Researchers like: Fuerst (2008), Norton (2009), Ceria and Stubbs (2006), Goldfarb and Iyengar (2003), Jorion (1992), Konno and Suzuki (1995), Michaud (1989a) (1989b), Bowen (1984), Ravipiti (2012) etc. discussed the weaknesses, limitations and assumptions in their works.

Since the discovering of the Markowitz's MV limitations and weaknesses, a lot of researchers have been working on the model to improve and develop it in different directions. Authors like Jobson, Korkie and

Ratti (1979), Jobson and Korkie (1980), Frost and Savarino (1988), Jorion (1992), Michaud (1998), (1989b), Polson and Tew (2000), etc. worked on the estimation error.

Others like Britten-Jones (2002), Kandel and Stambaugh (1996), Zellner and Chetty (1965), Klein and Bawa (1976) and Brown (1978) worked on the Markowitz's model by using Bayesian approach and predictive probability to improve and develop the model in various ways. Huang (2008) and Markowitz (1993) tried to develop the model to Mean-semi variance. Authors like Galluccio et al. (1998), Laloux et al. (1999), (2008), Bongini et al. (2002), Pafka and Kondor (2002), (2003), Potters et al. (2005), Lindberg (2009) and others brought in Random matrix theory (RMT), which was first proposed and introduced by Wigner (1951) and Laloux and Plerou introduced RMT in financial markets, to improve Markowitz's portfolio optimisation.

In this paper, we aim to optimize a portfolio containing stocks from the financial services sector of the Nigerian stock Exchange (NSE) using Markowitz's portfolio selection model and a three-objective linear programming model to allocate different percentage of weight to different assets to obtain an optimal/feasible portfolio, diversification of assets, and later we brought in cross - correlation of the individual stock of the sector to show the relationship between any two assets chosen in the correlation matrix. The rest of the paper is organised as follows. In section 2, we describe the nature of the empirical data used in the analysis. In section 3, we present the methodologies, theoretical background on mean-variance optimization; expected return and risk of the portfolio of the assets, constrain objective programme and cross - correlation of assets. Section 4, shows and discusses the main empirical result. Finally, section 5 concludes the paper.

2. Data

We obtained our data from NSE, which is made up of eleven (11) sectors, but our analysis is on the financial services. The financial services are about 57 assets from the time our analysis started. But we have to bring them down to 24 assets only in the course of our study. This development was necessary because some of assets were delisted from NSE due to some banks merging together and some bigger ones acquiring smaller one after the global melt down in order to meet up with the new capital base for the financial institutions operating in the country as ordered by the Central Bank of Nigeria (CBN). Also, some of the company under this sector did not trade more regularly within the time interval of our analysis and therefore, was removed. The data set used is the daily closing price of the stock data listed in the financial services of NSE. We have 1485 daily closing prices running from 3rd August 2009 to 4th August 2015, excluding weekends and public holidays in Nigeria (Nationwide). These stock price data were converted into 1484 logarithmic returns and was used in our analysis.

Let P_t be the closing price of the index on day t of stock and define the natural logarithmic returns of the index (i.e. the log-difference of P_{t+1} and P_t) as

$$r_t = \ln \left(\frac{P_{t+1}}{P_t} \right) \quad (1)$$

Where t has 1484 observation? Before establishing the portfolio selection process, we compute the mean return and standard deviation of each stock.

The table below shows the mean and standard variation of the individual stocks.

3. Theoretical background and methodology

3.1 Expected return of a portfolio.

The portfolio of n assets has each asset delivers a return of (r_i) at the time t . Each (r_i) has its mean and variance which is denoted as (μ_i) (σ_i^2) respectively. The money invested in the assets is regarded as weight of the asset (w_i) (which is less than 1 and sometimes a negative number is allowed if there is a short selling of any asset). Therefore, the summation of the individual weights of the assets that form the portfolio is 1, thus, $\sum w_i = 1$ and it is obvious to see that

$$\sum w_i = 1$$

Therefore,

$$w_i = \frac{1}{n} \quad (2)$$

and

$$(r_i) = [(r_i) - \mu_i] + \mu_i$$

Which implies that

$$(r_i) = \mu_i + [(r_i) - \mu_i] \quad (3)$$

,

...

Which is written as $(r_i) = \mu_i + [(r_i) - \mu_i]$, where μ_i = : and $[(r_i) - \mu_i]$ = : \therefore : are called the weight

... vector and covariance matrix respectively.

Let's recall that the correlation between any two assets is

$$= \frac{\text{Cov}(r_i, r_j)}{\sigma_i \sigma_j} \quad (4)$$

Where σ_i , σ_j are the standard deviation of r_i and r_j respectively, while ρ_{ij} is the coefficient of correlation of r_i and r_j , for $i, j = 1, 2, \dots, n$. The coefficient of correlation plays a great role in the portfolio diversification, if well managed; the coefficient of correlation will reduce the risk to a bearable level. In other words, the risk of a portfolio decreases as the coefficient of correlation of the assets moves from positive to negative.

Table 1

	AC CE SS	AI CO	CON TIN SUR E	CO RN ER ST	CUS TOD YIN S	DIA MON DBN K	FB N H	FC M B	FID ELI TYB K	GU AR BAN TY	MA NS AR D	NE M	NI GE RI NS	PR ES TI GE	RO YA LE X	SK YE BA NK	STE RLN BAN K	TRA NSC ORP	UA C- PR OP	UB A	UB N	W AP IC	WE MA BA NK	ZEN ITH BAN K
Av er ag e	- 0.00 21 8 %	- 0.00 22 9 %	- 0.02 84 %	- 0.03 33 4 %	0.02 53 %	- 0.05 10 %	- 0.00 60 6 %	- 0.00 64 0 %	- 0.00 40 9 %	0.0 335 %	- 0.0 14 0 %	0.4 51 8 %	- 0.0 73 6 %	- 0.1 56 6 %	- 0.0 44 7 %	0.1 84 4 %	0.02 35 %	0.0 987 %	- 0.0 23 5 %	0.0 8 1 %	- 0.0 68 3 %	0.0 04 7 %	0.01 82 %	0.00 58 %
Va ria nc e	0.00 68 6 %	0.01 03 6 %	0.08 78 %	0.03 33 8 %	0.09 71 %	0.07 85 %	0.00 54 6 %	0.00 69 2 %	0.0 779 %	0.0 577 %	0.0 85 9 %	3.1 96 3 %	0.0 41 6 %	0.0 82 4 %	0.0 67 7 %	1.1 02 9 %	0.09 60 %	0.1 124 %	0.0 83 6 %	0.2 02 2 %	0.5 12 2 %	0.2 60 1 %	0.21 48 %	0.06 05 %
Sta nd ar	2.6 18	3.2 19	2.96 37	1.8 39	3.11 64	2.80 21	2.3 36	2.6 30	2.7 915	2.4 023	2.9 31	17. 87	2.0 38	2.8 70	2.6 02	10. 50	3.09 76	3.3 522	2.84 90	4.4 96	7.15 56	0.4 99	6.3 48	2.46 06

respect to all feasible combinations. Our task now is to find that will maximize (,) with respect to all feasible combination

4. Empirical results and its analysis

Our main aim is to maximize the expected return and minimise the variance of the expected return of the portfolio containing assets from the financial services using the daily closing prices of the assets from 3rd of August 2009 to 4th of August 2015. This becomes 1484 days when all weekends and public holidays in Nigeria are excluded.

4.1 Portfolio1 Equally weighted Portfolio

We first constructed a portfolio that is equally weighted using the daily closing prices of the market, we got a portfolio which the return is 0.00162% and the standard deviation is 1.28% (see Table 2 and 3). Though, the standard deviation of the portfolio seems to be better than what we have from the market (see Table 1 and 4), but the return is very poor. In Table4 and figure 1, we can see that the single asset with the least risk is CORNERST, which is 1.84% but unfortunately, with a return that is very poor. Now our objective is to maximise the portfolio's return with a portfolio standard deviation which should be less than or equal to the least risk, (in other words we want construct a portfolio that the standard deviation will be less than or equal to that of CORNERST but the return will be above its return).

4.1 Portfolio 2: Maximization of the return

Therefore, we apply

$$(w_1, w_2, \dots, w_n) = \text{argmax} \quad (8)$$

Subject:

$$(w_1, w_2, \dots, w_n) = \frac{1}{n} \quad , \leq 1.84\%$$

$$(w_1, w_2, \dots, w_n) = \sum_{i=1}^n w_i - 1 = 0$$

Where w_i is the weight of individual assets, n is the number of the observations. After the simulation, the weights were distributed among the assets but assets like UBA, UBN, Diamond bank, ACCESS, FBNH, Fidelity bank, FCMB etc. were allocated with 0% of the weight while assets like Transcorp, Guaranty trust bank and Custody were given more percentage of the weight (see Table 4).

Table 2

		Portfolios		
	Equal Wt	Max Return	Min St Dev	Max SR
	None	at $\sigma \leq$	at $\mu =$	None
Value Constr	of N/a	1.840%	0.450%	N/a
ACCESS	4.1666%	0.0000%	0.0000%	0.0000 %

AIICO	4.1666%	0.0000%	0.0000%	0.0000%
CONTINSURE	4.1666%	0.0000%	0.0000%	0.0000%
CORNERST	4.1666%	0.0000%	0.0000%	0.0000%
CUSTODYINS	4.1666%	11.9229%	0.0000%	10.3272%
DIAMONDBNK	4.1666%	0.0000%	0.0000%	0.0000%
FBNH	4.1666%	0.0000%	0.0000%	0.0000%
FCMB	4.1666%	0.0000%	0.0000%	0.0000%
FIDELITYBK	4.1666%	0.0000%	0.0000%	0.0000%
GUARANTY	4.1666%	27.2599%	0.0000%	26.4968%
MANSARD	4.1666%	0.0000%	0.0000%	0.0000%
NEM	4.1666%	5.1779%	100.0000%	6.3839%
NIGERINS	4.1666%	0.0000%	0.0000%	0.0000%
PRESTIGE	4.1666%	0.0000%	0.0000%	0.0000%
ROYALEX	4.1666%	0.0000%	0.0000%	0.0000%
SKYEBANK	4.1666%	5.6436%	0.0000%	6.7855%
STERLNBANK	4.1666%	8.5136%	0.0000%	6.7090%
TRANSCORP	4.1666%	36.2113%	0.0000%	40.8552%
UAC-PROP	4.1666%	0.0000%	0.0000%	0.0000%
UBA	4.1666%	0.0000%	0.0000%	0.0000%
UBN	4.1666%	0.0000%	0.0000%	0.0000%
WAPIC	4.1666%	2.0102%	0.0000%	0.7009%

WEMABANK	4.1666%	2.9119%	0.0000%	1.7415%
ZENITHBANK	4.1666%	0.3488%	0.0000%	0.0000%
Σw	100.00%	100.00%	100.00%	100.00%
μ_p	0.00162%	0.0839%	0.450%	0.0946%
σ_p	1.281%	1.840%	17.89%	2.060%
μ/σ	0.126%	4.56%	2.52%	4.59%

Table of our four different portfolios constructed.

Though in this new portfolio, we got a standard deviation that is greater than that of the portfolio with equal weighted assets, but the return is very encouraging. The return is about 52 times of the return of the said portfolio (see Table 4). Again, if we look at the return of the asset with the least standard deviation (CORNERST with $\sigma = 1.84\%$, see Table 3), you will notice that it cannot be compared to our new return. Finally, if we look at the Sharpe ratio (SR) of the portfolios, SR of the equal weighted portfolio and our new portfolio are 0.12% and 4.56% respectively (see Table 4) and the stock with the least standard deviation has its SR to be 1.81% (Table 2), this shows that 4.56% is best among all.

4.3 Portfolio 3: Minimization of Standard Deviation

4.4

In this case, we want to minimise the standard deviation of the single asset with maximum SD (NEM) which is 17.88% (Table 1), to see if we will get a lower SD and an improved return (which may not necessarily be equal to the return of the said asset). Therefore, we apply

$$\left(\begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} \right) = \frac{1}{-1} \quad (9)$$

Subject to

$$\left(\begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} \right) = - \geq 0.450\%$$

$$\left(\begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} \right) = -1 = 0$$

After the simulation, we got a funny result where 100% of our weight is allocated to NEM, with return and SD equal to what we had abinitio and therefore this portfolio is not acceptable.

4.5 Portfolio 4: Maximization of Sharpe ratio

Finally, we maximise the sharpe ratio SR. Here we have the equation as follows

$$\left(\begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} \right) = \quad (10)$$

$$\left(\begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{matrix} \right) = -1 = 0$$

Again, we have the return to be 0.095%, the SD to be 2.06% and SR 4.6%. The weights were loaded in Transcorp, Guaranty trust bank and Custody assets with very few distributed among Skye, Sterling and Wema Banks, others are Wapic and NEM.

Comparison of the results, that is, equally weighted portfolio, Max. Return, Min. Standard deviation and Max

SR as shown in Table 3

	Portfolios			
	Equal Wt	Max Return	Min St Dev	Max SR
μ_p	0.00162%	0.084%	0.45%	0.095 %
σ_p	1.28%	1.84%	17.89%	2.06 %
μ/σ	0.13%	4.56%	2.52%	4.60%

Table 3.A table showing the return, risk and sharpe ratio of the four portfolios constructed.

If we take the equally weighted portfolio as our pivotal portfolio, with return, standard deviation and sharpe ratio as 0.00162%, 1.28% and 0.13% respectively, we notice that it return was below expectations. Though the risk is very minimal but the return and the sharpe ratio show that it is not a good idea to invest in the sector with an equally weighted portfolio. The portfolio that minimizes standard deviation has the highest return but the risk is too much and the sharpe ratio is not encouraging, also the simulation allocated 100% of the weight to one stock (NEM) which does not encourage diversification of funds. Therefore, these make it not healthy for investment. We are now left with two options which are, Max Return and Max SR which have their returns as multiples of 52 and 59 of the return of the equally weighted portfolio respectively. Though the risk value of both is greater than the value of the equally weighted portfolio but the sharpe ratios are better, which again are multiples of 46 on approximate of the equally weighted portfolio.

Table 4.

Individual	Assets			
	Average	Variance	Standard D	μ/σ
ACCESS	- 0.022%	0.069%	2.620%	-0.832557 %
AIICO	- 0.023%	0.100%	3.220%	-0.711739 %
CONTINSURE	- 0.028%	0.088%	2.960%	-0.960405 %
CORNERST	- 0.033%	0.034%	1.840%	-1.812935 %
CUSTODYINS	0.025%	0.097%	3.120%	0.809808 %

DIAMONDBNK	-0.051%	0.079%	2.800%	-1.821179%
FBNH	-0.061%	0.055%	2.340%	-2.589444%
FCMB	-0.064%	0.069%	2.630%	-2.431863%
FIDELITYBK	-0.041%	0.078%	2.790%	-1.464946%
GUARANTY	0.033%	0.058%	2.400%	1.394083%
MANSARD	-0.014%	0.086%	2.930%	-0.478976%
NEM	0.450%	3.200%	17.880%	2.516779%
NIGERINS	-0.074%	0.042%	2.040%	-3.609265%
PRESTIGE	-0.160%	0.082%	2.870%	-5.574913%
ROYALEX	-0.045%	0.068%	2.600%	-1.718692%
SKYEBANK	0.180%	1.100%	10.500%	1.714286%
STERLNBANK	0.024%	0.096%	3.100%	0.758484%
TRANSCORP	0.099%	0.110%	3.350%	2.945343%
UAC-PROP	-0.023%	0.084%	2.890%	-0.813114%
UBA	0.008%	0.200%	4.500%	0.181098%
UBN	-0.170%	0.510%	7.160%	-2.374302%
WAPIC	0.005%	0.260%	5.100%	0.092069%
WEMABANK	0.018%	0.210%	4.630%	0.392613%
ZENITHBANK	0.006%	0.061%	2.460%	0.237573%

Table showing individual assets return, risks and sharpe ratio from financial sector of NSE
Fig.1



Histogram showing the return, risk and sharpe ratio of the constructed portfolios.

Fig 2.

	AC CES S	AII CO	CO NT IN SU T	CO RN ERS OD YI T	CU ST OD YI D	DI AM ON D	FB NH	FC MB	FID ELI TY BB	KG UA RA NT	MA NS AR D	NE M	NI GE RI NS	PR ES TIG E	RO YA LE X	SKY EB AN K	STE RL NB A	TR AN SC O	RU AC PR O	UB A	UB N	W AP IC	W EM AB A	NZE NIT HBA N
AC CES S	1	0.1 50 76	0.0 56 04 1	- 0.0 112 9	0.0 54 67 5	0.2 39 82 5	0.1 95 36 9	0.1 27 06 6	0.1 874 93	0.1 657 77	0.0 24 93 3	0.0 22 42 5	0.0 17 90 9	0.0 13 76 8	0.0 62 96 1	- 0.0 018 1	0.1 278 25 3	- 0.0 25 29	0.0 30 21 9	0.0 67 86 9	0.0 02 77 8	0.0 53 06 3	0.0 16 86 1	0.22 352 1
AII CO	0.1 507 6	1	0.0 31 98 2	0.0 099 87	0.0 24 21 1	0.0 67 55 4	0.1 10 69 4	0.0 61 03 4	0.0 436 42	0.1 327 21	- 0.0 25 68	0.0 06 56 3	0.0 15 02 8	0.0 00 71 7	0.0 06 87 3	- 0.0 015 4	0.0 825 10 36	- 0.0 10 36	0.0 02 98 9	0.0 22 63 6	- 0.0 47 42	0.0 18 64 2	0.0 45 98 3	0.12 053 3
CO NTI NS U	0.0 560 41	0.0 31 98 2	1	0.0 614 41	- 0.0 75 55	0.0 13 91 8	0.0 44 60 2	- 0.0 01 33	0.0 785 3	0.0 162 89	0.0 20 51 5	0.0 10 00 1	0.0 17 14 7	- 0.0 46 02	- 0.0 29 66	0.0 111 93	0.0 038 15	0.0 50 33 8	- 0.0 02 12	0.0 36 77 8	0.0 07 53 8	0.0 10 73 8	- 0.0 11 25	0.03 104 8
CO RN ERS	T 0.0 112 9	- 0.0 09 7	0.0 61 44 1	1	0.0 05 12 74	0.0 08 55 8	0.0 07 37 6	0.0 57 54 8	0.0 148 8	- 0.0 207 8	- 0.0 20 35	0.0 06 91 8	0.0 15 99 5	- 0.0 25 65	- 0.0 28 34	0.0 136 71	0.0 326 36	0.0 06 60 2	0.0 58 52 6	- 0.0 11 41	0.0 01 81 2	0.0 38 14 1	0.0 24 66 1	- 0.02 996
CUS TO DYI	0.0 546 76	0.0 24 21 2	- 0.0 75 55	0.0 127 4	1	0.0 49 43 1	0.0 40 05 7	0.0 45 62 1	0.0 376 37	0.0 464 67	0.0 41 19	0.0 02 37	0.0 14 38 7	0.0 07 51 7	- 0.0 09 12	0.0 073 32	0.0 721 69	0.0 05 14	0.0 39 7	0.0 29 58 8	0.0 11 62	0.0 10 10 2	0.0 18 73 2	0.05 348 7
DIA MO ND	0.2 398 25	0.0 67 55 1	0.0 13 91 8	0.0 055 58	0.0 49 43 1	1	0.1 74 68 6	0.0 97 73 4	0.2 903 97	0.1 467 93	- 0.0 24 85	0.0 00 14 1	- 0.0 20 2	0.0 61 03 2	0.0 31 77 1	0.0 656 86	0.1 420 97	0.0 01 91 6	0.0 14 64	0.1 59 34	- 0.0 08 86	0.1 02 07 7	0.0 59 74 5	0.14 502 8
FB NH	0.1 953 69	0.1 10 69 4	0.0 44 60 2	0.0 083 76	0.0 40 05 7	0.1 74 68 6	1	0.1 17 73 2	0.1 423 78	0.3 727 94	0.0 28 78 8	0.0 05 89 7	0.0 01 80 5	- 0.0 70 33	0.0 13 71 2	- 0.0 135 5	0.1 188 83	- 0.0 33 32	0.0 80 75 1	0.0 91 05 9	- 0.0 22 9	0.0 20 16 3	0.0 39 20 1	0.38 548 6

FC	0.1	0.0	-	0.0	0.0	0.0	0.1	0.1	0.0	0.0	-	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	0.11
MB	270	03	01	575	62	73	73	065	121	50	27	03	27	42	354	643	91	26	34	41	03	852
	66	4	33	48	1	4	2	1	94	51	1	5	42	13	7	81	12	7	3	3	52	4
FID	0.0	0.0	0.0	0.0	0.2	0.1	0.1	0.1	0.0	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ELI	0.1	43	0.0	0.0	37	90	42	06	0.1	21	0.0	13	45	88	0.0	0.1	06	15	97	13	80	73
TY	874	64	78	148	63	39	37	59	406	66	00	38	13	69	557	170	32	78	52	02	51	281
B	93	2	53	8	7	7	8	4	1	85	8	39	6	1	1	57	26	8	3	4	1	7
GU	0.1	0.0	-	0.0	0.1	0.3	0.1	0.1	0.0	0.0	-	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AR	0.1	32	16	0.0	46	46	72	12	0.1	08	00	0.0	0.0	0.0	0.0	0.1	0.0	68	75	05	53	28
AN	657	72	28	207	46	79	79	15	406	12	97	26	03	17	051	084	43	89	79	19	23	26
T	77	1	9	8	7	3	4	1	85	1	2	6	38	31	57	44	29	27	8	8	3	9
MA	-	0.0	-	-	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	0.0	0.0	0.0
NS	0.0	0.0	20	0.0	0.0	0.0	28	29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	16	0.0	27	43	07
AR	249	25	51	203	41	24	78	50	216	081	89	37	97	47	006	037	64	21	62	36	49	02
D	33	68	5	5	19	85	8	1	68	22	1	6	6	3	3	86	3	6	54	2	7	1
NE	0.0	0.0	0.0	-	0.0	0.0	0.0	-	0.0	0.0	0.0	-	0.0	-	0.0	0.0	-	-	0.0	-	-	0.0
M	224	56	00	069	02	14	89	27	003	009	89	72	05	87	044	048	13	08	02	00	01	00
	25	3	1	18	37	1	7	5	9	76	6	1	6	39	9	4	99	98	52	5	15	28
NIG	0.0	0.0	0.0	0.0	-	0.0	-	-	0.0	0.0	0.0	0.0	-	0.0	-	-	-	0.0	0.0	0.0	0.0	0.0
ERI	179	02	14	159	38	20	80	03	133	263	37	72	13	21	010	018	15	46	31	74	05	31
NS	09	8	7	95	7	2	5	42	86	8	6	6	1	1	36	65	2	68	1	1	4	6
PR	0.0	0.0	-	-	0.0	0.0	-	-	0.0	0.0	-	0.0	0.0	0.0	0.0	-	-	-	-	-	0.0	-
EST	137	71	46	256	51	03	70	27	451	033	97	05	13	42	028	030	18	35	26	21	43	14
IGE	68	7	02	5	7	2	33	13	31	1	3	39	1	1	5	79	8	09	82	24	89	2
RO	0.0	0.0	-	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	0.0	0.0	0.0	0.0	-	0.0	0.0	0.0	0.0
YA	629	87	29	283	09	77	71	42	886	175	47	87	21	42	129	674	11	12	73	69	58	49
LEX	61	3	66	4	12	1	2	7	91	7	3	9	36	5	1	32	54	2	78	1	7	2
SKY	-	-	0.0	0.0	0.0	0.0	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	-	-	0.0
EB	0.0	0.0	11	0.0	07	65	0.0	35	0.0	0.0	0.0	0.0	01	02	12	0.0	29	24	53	0.0	0.0	19
AN	018	01	19	136	33	68	13	48	557	051	68	04	06	87	93	258	05	99	35	10	02	21
K	1	54	3	71	2	6	55	1	57	44	6	44	5	9	2	1	61	7	6	9	09	2
STE	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.0	0.0	-	0.0	-	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RL	0.1	82	03	0.0	72	42	18	64	0.1	0.1	0.0	04	0.0	0.0	67	0.0	06	70	0.0	08	06	74
NB	278	53	81	326	16	09	88	31	170	084	03	89	01	03	45	258	65	40	38	22	26	59
A	3	6	5	36	9	7	3	2	26	29	73	9	82	08	4	61	1	9	7	45	3	9
TR	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AN	R	-	0.0	066	-	0.0	-	0.0	063	-	0.0	-	-	-	0.0	290	066	0.0	0.1	0.0	-	25
SCO	0.0	0.0	50	02	0.0	01	0.0	59	28	0.0	16	0.0	0.0	0.0	18	57	59	1	22	04	00	63

	2529	1036	338		0514	916	3332	917		4327	646	1398	1568	1809	112				529	056	208	1773		
UA	0.0	0.0	0.0	0.0	0.0	0.0	80	72	0.0	0.0	-	-	0.0	-	-	0.0	0.0	22		-	0.0			
CP	302	98	02	585	39	14	75	26	157	688	21	08	46	35	12	249	704	52		0.0	0.0	57	0.0	
RO	19	9	12	26	7	64	1	3	83	98	54	52	1	82	78	96	07	9	1	57	04	1	92	0.02
		0.0	0.0	-	0.0		0.0	0.0			0.0	0.0	0.0	-	0.0		0.1			-	0.0	0.0		
UB	0.0	0.22	0.36	0.0	0.29	0.1	0.91	0.75	0.0	0.0	0.27	0.34	0.15	0.0	0.02	0.7	0.0	0.04	0.0		0.0	0.59	0.34	
A	678	63	77	114	58	59	05	34	975	757	62	02	31	26	73	533	384	05	35		0.0	0.23	0.95	
	69	6	8	1	8	34	9	3	24	98	2	5	1	24	1	59	5	6	57	1	88	7	0.09	
		-	0.0			-	-	-			0.0	-	0.0	-	0.0	-		0.0	-	-		0.0	-	
UB	0.0	0.0	0.07	0.0	0.0	0.0	0.0	0.0	0.0	0.0	43	0.0	19	0.0	0.02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	
N	027	47	53	018	11	08	22	41	130	051	36	00	74	21	69	100	082	20	22	23		0.09	0.03	
	78	42	8	12	62	86	9	52	21	93	7	15	4	89	7	9	23	8	04	88	1	7	381	
		0.0	0.0		-	0.1	0.0				0.0	-	0.0	0.0	0.0			-	0.0	0.0	0.0		0.0	
WA	0.0	0.18	0.10	0.0	0.0	0.02	0.20	0.0	0.0	0.0	0.07	0.0	0.49	0.14	0.11	-	0.0	0.0	0.57	0.59	0.04		0.12	
PIC	530	64	73	381	10	07	16	03	805	532	49	01	05	43	58	0.0	062	17	53	05	09		0.80	
	63	2	8	41	2	7	3	4	12	39	1	28	6	2	2	022	69	73	1	7	7	1	957	
		0.0	-		0.0	0.0	0.0	-			0.0	0.0	0.0	-	0.0				0.0	-	0.0		0.07	
WE	0.45	0.0	0.0	0.0	0.18	0.59	0.39	0.0	0.0	0.0	0.42	0.07	0.43	0.0	0.0	0.0	0.0	0.0	0.0	0.34	0.0	0.12		
MA	0.16	0.98	0.11	0.246	0.73	0.74	0.20	0.03	0.737	0.282	0.02	0.00	0.31	0.14	0.49	0.192	0.745	0.25	0.20	0.95	0.03	0.80	0.07	
BA	0.861	0.3	0.25	0.61	0.2	0.5	0.1	0.82	0.97	0.67	0.8	0.8	0.9	0.86	0.5	0.19	0.96	0.63	0.92	0.5	0.39	0.8	0.301	
ZE		0.1	0.0	-	0.0	0.1	0.3	0.1				-	0.0	0.0	0.0			-		-	0.0	0.0		
NIT	0.2	0.20	0.31	0.0	0.53	0.45	0.85	0.18	0.2	0.3	0.0	0.0	0.18	0.15	0.38	0.0	0.1	0.0	0.0	0.0	0.69	0.73		
HB	235	53	04	299	48	02	48	52	228	577	63	07	14	71	32	195	288	55	20	99	03	57	0.01	
A	21	3	8	6	7	8	6	4	17	04	49	11	5	1	6	01	94	26	97	79	81	3	1	

The correlation matrix of the assets in the financial sector of NSE

5. Conclusion

We were able to construct four portfolios as we can see the summery in Table 3. The first one is equally weighted and the return is so small that an investor who wants a profit will not be advised to invest in such portfolio.

Secondly, the portfolio that was formed with equation (9) gave the highest return with a very high standard deviation which is not encouraging. Besides, the idea of diversification was killed because the whole fund was allocated to one stock which is NSE. Therefore we advise investors to disregard this. Finally, the equations (8) and (10) gave us something closer to what we want, an appreciable return and a risk that can be tolerated and above all, their sharpe ratios are within acceptable boundaries when compared with the former two. Investors who want to invest in this sector are advised to invest in the portfolio of equation (10), which we consider to be the optimal portfolio. Though, the risk is slightly above the other but the return and the sharpe ratio are very encouraging. Furthermore, we can see from the interaction of the stocks in the correlation matrix (fig 2), that the assets selected in the portfolio move in such direction that will reduces risk. Investors are highly advised to invest in this portfolio.

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