Entrepreneurship, Business and Economics Research Journal
ISSN: 2997-6782 |
Volume 11 Issue 1, January-March, 2023
Journal Homepage: https://ethanpub.online/Journals/index.php/E14
Official Journal of Ethan Publication

DECIPHERING THE FORMULA: SIMPLIFYING THE UNDERSTANDING OF HOLDING PERIOD RETURNS

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Abstract:
The concept of holding period return (R) is a fundamental measure in finance, representing the ratio of future proceeds to the initial investment. For bonds, this calculation is defined as R = (B1 – B0 + iF)/B0, where Bt denotes the bond valuations at time t, iF represents interest payments at the interest rate i on face value F, and M signifies maturity, discounted at rate k, known as the yield to maturity. Corporate bonds often entail semi-annual interest payments, equivalent to half the annual iF amount. These interest payments can be conceptualized as an annuity, iF/k(1– 1/[1+k]M), while the face value is F/(1+k)M. This abstract delves into the mathematical intricacies of holding period returns for bonds and provides insights into their underlying principles.

Keywords: Holding Period Return, Bond Valuation, Yield to Maturity, Corporate Bonds, Interest Payments

Introduction

A holding period return R is a ratio of future proceeds divided by its initial investment. For a bond it is $R = (B_1 - B_0 + iF)/B_0$ with bond valuations B_t at time t with interest payments of iF at interest rate i on face value F, and with a maturity M discounted at rate k which for bonds is called the yield to maturity. Interest payments on corporate bonds are often paid twice a year as half the annual amount of iF. The interest payments are an annuity as $iF/k(1-1/[1+k]^M)$ and the face value is $F/(1+k)^M$ or:

$B_0 = iF/k(1-1/[1+k]^M) + F/(1+k)^M$ and $B_1 = iF/k(1-1/[1+k]^{M-1}) + F/(1+k)^{M-1}$. Thus:

$R = \{iF/k(1-1/[1+k]^{M-1})+F/(1+k)^{M-1}- iF/k(1-1/[1+k]^M) - F/(1+k)^M+iF\}/\{iF/k(1-1/[1+k]^M)+F/(1+k)^M\}$, canceling F:

$R = \{i/k(1-1/[1+k]^{M-1})+1/(1+k)^{M-1}- i/k(1-1/[1+k]^M) -1/(1+k)^M +i\}/ \{i/k(1-1/[1+k]^M)+1/(1+k)^M\}$, expanding the annuities:

$R = \{i/k-[i/k]/[1+k]^{M-1}+1/(1+k)^{M-1} -i/k+[i/k]/[1+k]^M -1/(1+k)^M +i\}/ \{i/k -[i/k]/[1+k]^M+1/(1+k)^M\}$, canceling like terms:

$R = \{-[i/k]/[1+k]^{M-1}+1/(1+k)^{M-1}+[i/k]/[1+k]^M- 1/(1+k)^M+i\}/ \{i/k -[i/k]/[1+k]^M+1/(1+k)^M\}$, multiplying by $(1+k)^M$:

$R = [-i(1+k)/k+(1+k)+i/k -1 +i(1+k)^M]/[i(1+k)^M/k -i/k+1]$, and multiplying by k:

$R = [-i(1+k)+(1+k)k+i -k +ik(1+k)^M]/[i(1+k)^M-i+k]$, and expanding:

$R = [-i -ik+k+kk+i -k +ik(1+k)^M]/[i(1+k)^M-i+k]$, and canceling:

$R = [-ik +kk +ik(1+k)^M]/[i(1+k)^M-i+k]$.

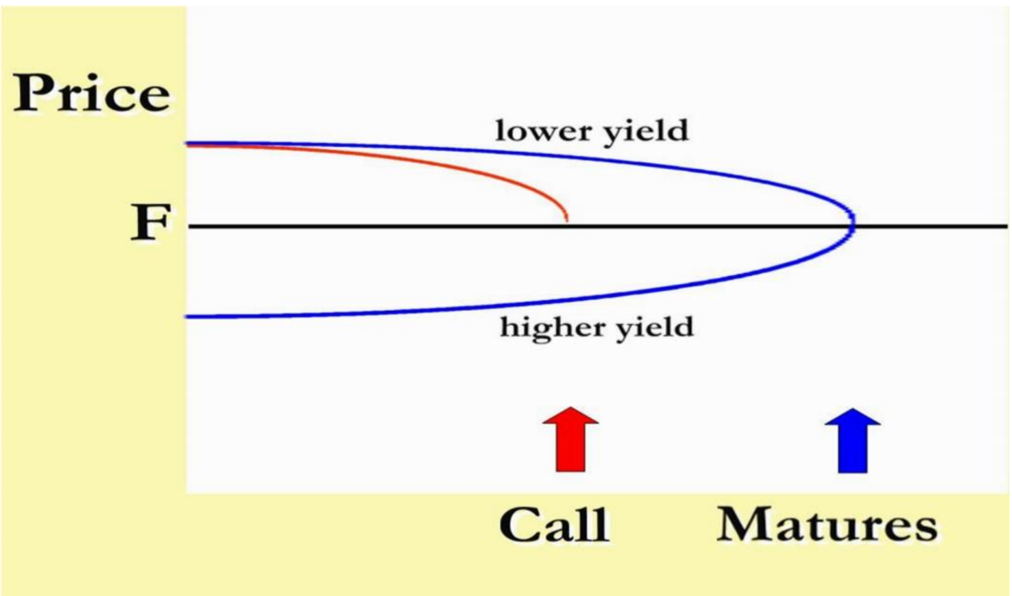
The numerator is a multiple of the denominator by k, therefore R = k.

Here are some different examples of holding period returns for various bonds discounted at a 10 percent yield: Coupon

Interest Maturity Price Maturity Price Yield

0 2 86.64 1 90.91 10
1 3 77.62 2 84.38 10
5 30 52.87 29 53.15 10
10 20 100.00 19 100.00 10
12 5 107.58 4 106.34 10 15 20 142.56 19 141.82
10

Do be wary of bonds trading at a premium and/or convertible bonds. For bonds trading at a premium, yield to call price and call date would be more appropriate.



For a common stock the holding period return R with valuations P_t and D_t at time t, the yearly return would be $R = (P_1 - P_0 + D_1)/P_0$. Often dividends are paid four times a year with an associated decrease in value on its exdividend date; exchange traded funds often pay monthly. A common stock with a dividend payment of D_t and discounted at rate k with growth rate g, is valued as:
$P_0 = D_1/(k-g) = D_0(1+g)/(k-g)$;
Substituting: $R = (D_1[1+g]/[k-g] - D_1/[k-g] + D_1)/(D_1/[k-g])$, Canceling D_1:
$R = ([1+g]/[k-g] - 1/[k-g] + 1)/(1/[k-g])$, Multiplying by (k−g): $R = (1 + g - 1 + k - g)$ Which results in $R = k$.
Here are some examples of different common stocks with different dividends and different growth rates discounted at a 10 percent discount rate:

Growth Current Current Next Next
Rate Dividend Price Dividend Price Yield

0 2.00 20.00 2.00 20.00 10
4 1.00 17.33 1.04 18.03 10
6 4.00 106.00 4.24 112.36 10 8 1.00 54.00 1.08 58.32 10

Do be wary that tax considerations were neutral here whereas in reality capital gains tax rates and the tax rates upon dividends may differ.

Note that $D_1 = E_1 (1\text{-}b)$ where E is a firm's earnings, b is the firm's retention rate, and a firm's endogenous growth rate g may be determined by g = br where r is the firm's rate of return. Earnings are achieved on the firm's assets A or $E_1 = A_0r$.

Thus:

$P_0 = D_1/(k\text{-}g) = E_1(1\text{-}b)/(k\text{-}g) = A_0r(1\text{-}b)/(k\text{-}br)$.

Likewise $k = D_1/P+g = E_1(1\text{-}b)/P+br = A_0r(1\text{-}b)/P+br$. Where r is greater (less) than k, P will be valued greater (less) than A. However, when r is greater than k, a lesser dividend and a greater retention rate resulting in increased growth will increase valuations, whereas when r is less than k then a greater dividend and lesser retention and lower growth rates will increase valuations albeit still below asset valuation A. Consider A_0 equaling 100, with r equaling .12 and E_1 equaling 12, the valuations are:

b	1-b	g	D_1	k=.14	k=.12	k=.10
0	1	.00	12	85.7	100.0	120.0
¼	¾	.03	9	81.8	100.0	128.6
½	½	.06	6	75.0	100.0	150.0
¾	¼	.09	3	60.0	100.0	300.0

When r equals k, P equals A validating the Miller and Modigliani proposition that dividends do not matter. A shortcut to a payout ratio is dividend yield times the P-E ratio or (D/P)(P/E) = D/E = (1-b). A valuation using the price-earnings ratio follows from $P_0 = E_1(1\text{-}b)/(k\text{-}br)$ where P-E = (1-b)/(k-br) and when multiplied by the expected earnings E_1 provides a stock valuation. In equilibrium when k equals r, the P-E ratio equals 1/k.

Conclusion

While obvious once the mathematics are examined, I repeatedly ask my students what is the holding period return to bonds and stocks and rarely get a correct response. Thus I'm repeatedly reminded that this exercise is well worth the review.

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